Laplacian Renormalization Group: A 'zoom lens' for networks

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(Dated: June 14, 2023)

Complex networks usually exhibit a rich architecture organized over multiple intertwined scales. Information pathways are expected to pervade these scales reflecting structural insights that are not manifest from analyses of the network topology. Moreover, small-world effects correlate the different network hierarchies complicating the identification of coexisting mesoscopic structures and functional cores. We present a communicability analysis of effective information pathways throughout complex networks based on information diffusion to shed further light on these issues. This will lead us to a formulation of a new and general renormalization group scheme for heterogeneous networks. The Renormalization Group is the cornerstone of the modern theory of universality and phase transitions, a powerful tool to scrutinize symmetries and organizational scales in dynamical systems. However, its network counterpart is particularly challenging due to correlations between intertwined scales. To date, the explorations are based on hidden geometries hypotheses. Here, we propose a Laplacian RG diffusion-based picture for complex networks, defining both the supernodes concept à la Kadanoff, the equivalent momentum space procedure à la Wilson for graphs, and applying this RG scheme to real networks in a natural and parsimonious way.

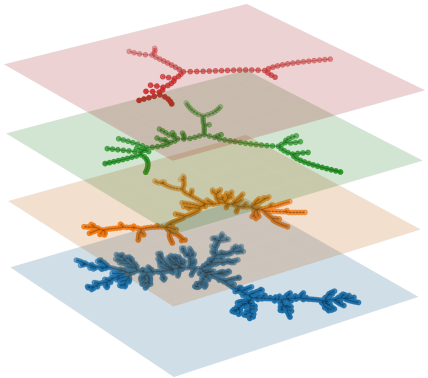

FIG. 1. Laplacian Renormalization Group procedure for a random tree.

[1] Villegas, P., Gili, T., Caldarelli, G. et al. Laplacian renormalization group for heterogeneous networks. Nat. Phys. **19**, 445–450 (2023).
[2] Klemm, K. A zoom lens for networks. Nat. Phys. **19**, 318–319 (2023)
[3] Multi-scale Laplacian community detection in heterogeneous networks. arXiv:2301.04514, (2023).

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