Optimal representations in neural systems at the edge of chaos

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Shedding light on how biological systems represent, process and store information in noisy environments is a key and challenging goal. A stimulating yet controversial hypothesis poses that operating in dynamical regimes near the edge of a phase transition, i.e., at criticality or the "edge of chaos", can provide information-processing living systems with important operational advantages, creating an optimal trade-off between robustness and flexibility. Here, we elaborate on a recent theoretical result, which establishes that the spectrum of covariance matrices of neural networks representing complex inputs in a robust way needs to decay as a power-law of the rank, with an exponent that depends on the dimensionality of the input, a result that has been indeed experimentally verified in neurons of the mouse visual cortex. Aimed at understanding and mimicking these results, we construct an artificial neural network and train it in two different types of tasks: image classification and timeseries prediction. We find that the best performance in such tasks is obtained when the network operates near the critical point, at which the eigenspectrum of the covariance matrix follows the very same dependence with the dimensionality of the input as actual neurons do. Thus, we conclude that operating near criticality can also have—besides the usually alleged virtues—the advantage of allowing for flexible, robust and efficient input representations.

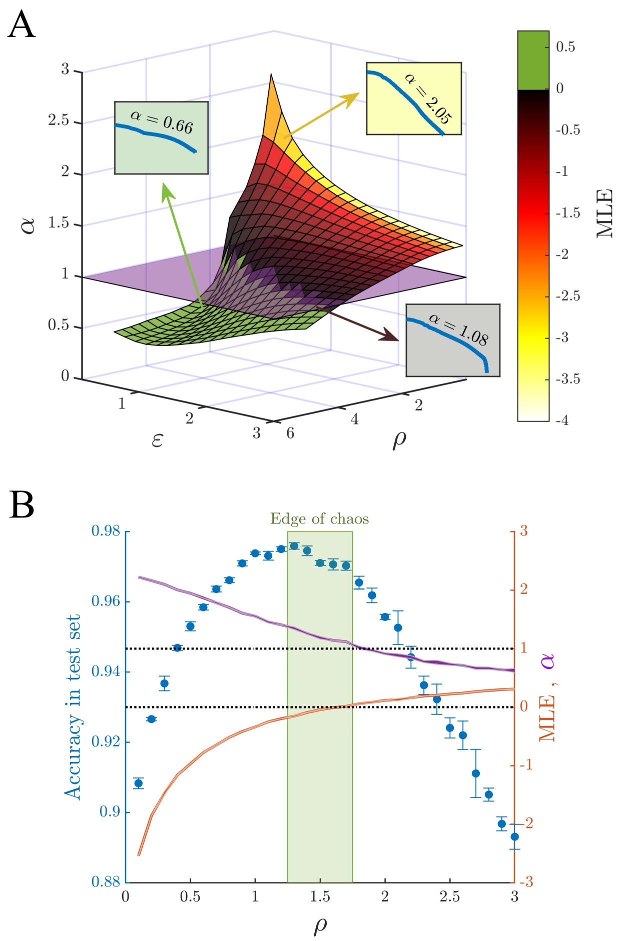

Fig. 1. **A**. Exponent for the power-law decay of the spectrum of the activity covariance matrix as a function of the spectral radius (ρ) and input scaling factor (ε) of the reservoir, plotted together with the maximum Lyapunov exponent (MLE) color-coded within the surface. The insets correspond to the activity covariance matrix eigenspectrum measured in three different points of the parameter space, where the variance in the n-th dimension (n-th eigenvalue) scales as a power-law $n^{-\mu}$ of the rank. The purple plane marks the boundary $\mu = 1$ for smooth representations of high-dimensional inputs. **B**. Accuracy in MNIST testset (blue dots), maximum Lyapunov exponent (orange line) and best fit exponent for the power-law spectrum of the activity covariance matrix (purple line).

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