SPECTRAL QUANTIZATION OF DISCRETE RANDOM WALKS ON HALF-LINE, AND ORTHOGONAL POLYNOMIALS ON THE UNIT CIRCLE

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We define quantization scheme for discrete-time random walks [4] on the half-line consistent with Szegedy's quantization[5] of finite Markov chains. Our procedure agrees with Szegő map [6] between orthogonal polynomials on the circle and on segment of the real line. It can be applied to arbitrary discrete-time Karlin and McGregor random walks [3] generalizing the so called CGMV method [1] for quantum walks. We illustrate our approach by quantization of random walks with constant transition probabilities where the corresponding polynomials on the unit circle have two-periodic Verblunsky coefficients. We comment on relation of the discussed quantum walks to integrable geometry of discrete curves [2].

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