Anomalous epidemic spreading

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16th Granada Seminar:
New Frontiers in Nonequilibrium Physics
Granada, June 7-17, 2021

Epidemics

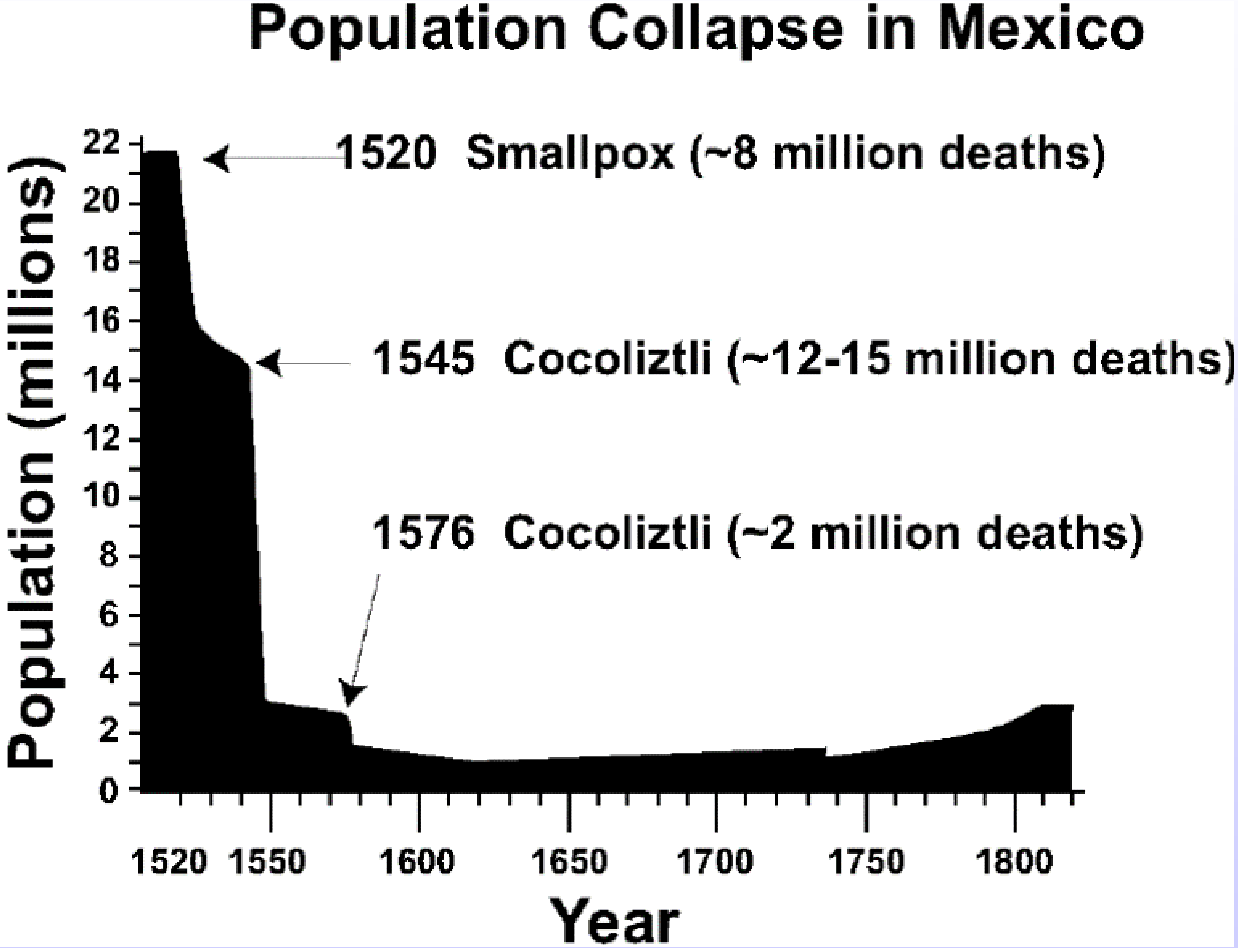
humans

fish

cows

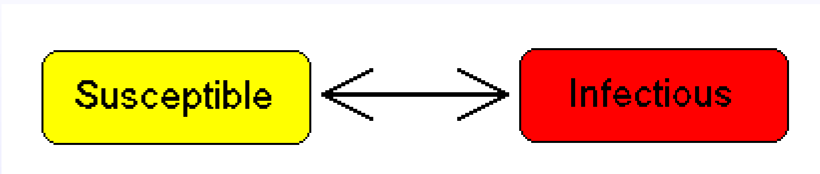
olive trees → square lattice

Cost of Epidemics

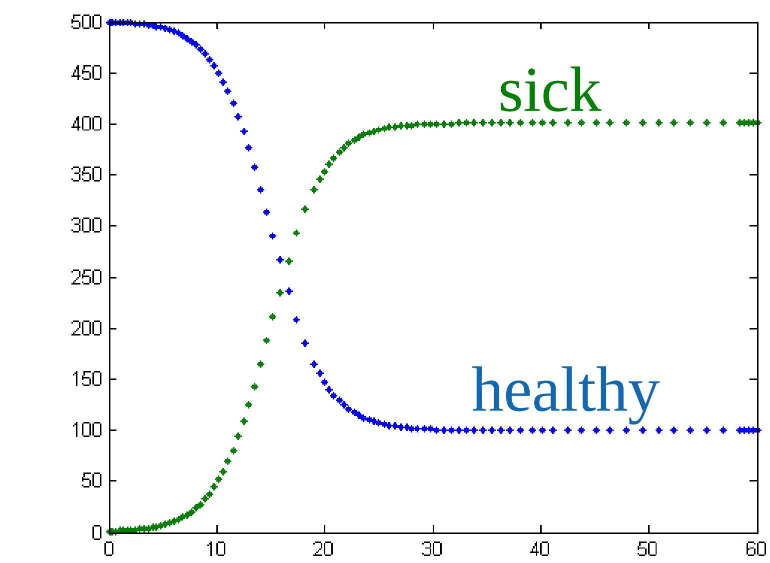


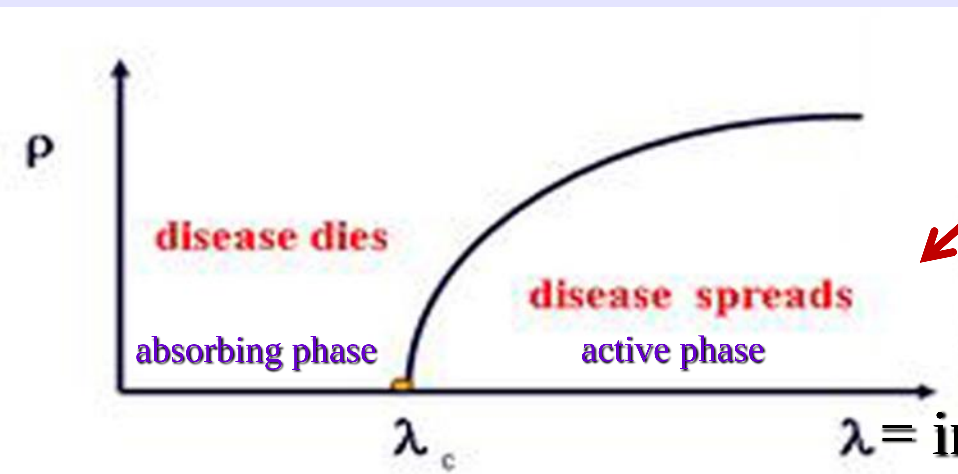
SIS Model for Epidemics

Daniel Bernoulli 1760



$$S \xrightarrow{p} I \xrightarrow{q} S$$





λ = infection rate

$$\lambda = k \frac{p}{q}$$

k = number of contacts of each person.

Mean Field of SIS Model

$$\frac{ds}{dt} = q\, i(\mathrm{t}) - kp\, s(\mathrm{t})\, i(\mathrm{t})$$

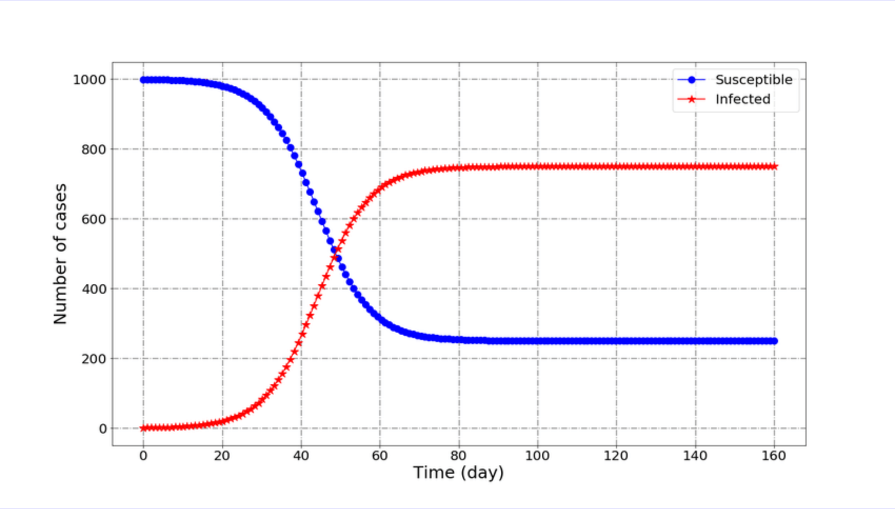
$$\frac{di}{dt} = -\, q\, i(\mathrm{t}) + kp\, s(\mathrm{t})\, i(\mathrm{t})$$

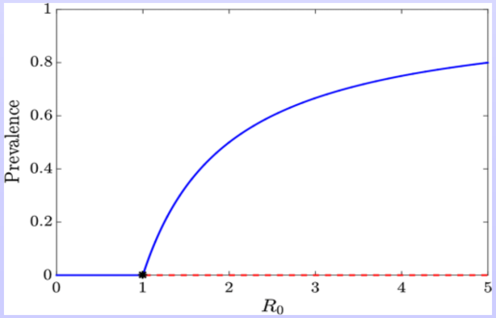
s(t) is fraction of susceptible agents
i(t) is fraction of infected agents

$$s(\mathrm{t}) + i(\mathrm{t}) = 1$$

$$\frac{di}{dt} = (kp - kp\, i(\mathrm{t}) - q)\, i(\mathrm{t})$$

$$\frac{di}{dt} = 0 \;\rightarrow\; i(\infty) = 1 - {}^{1}/_{\lambda}$$





Classical Variants

SIS applies to sexually transmitted diseases
as well as most bacterial and parasitic infections.

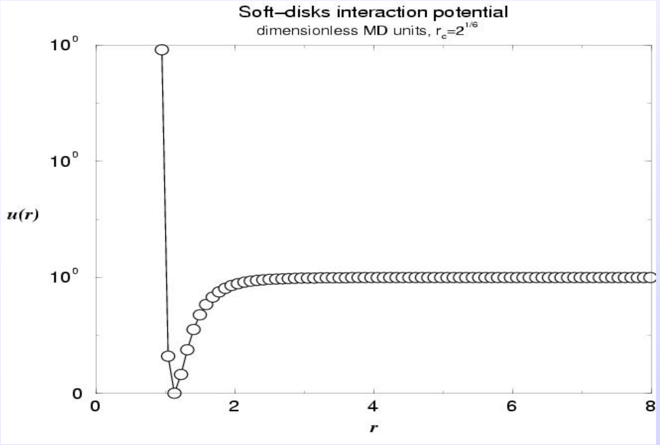
- include immunization: SIR model
- include latent period (delay)
- include chronical infections
- include waning immunity: SIRS model
- include births and natural deaths
- include population structure
- include transmission through vector

Agent based SIS Model for Epidemics

Marta González

soft mobile agents in two dimensions

Lennard Jones interaction potential



$$u(r_{ij}) = U_0\left[\left(\frac{\sigma}{r_{ij}}\right)^{12} - \left(\frac{\sigma}{r_{ij}}\right)^{6}\right] + U_0 \quad , \quad r_{ij} \le r_c = 2^{1/6}\sigma$$

collision time

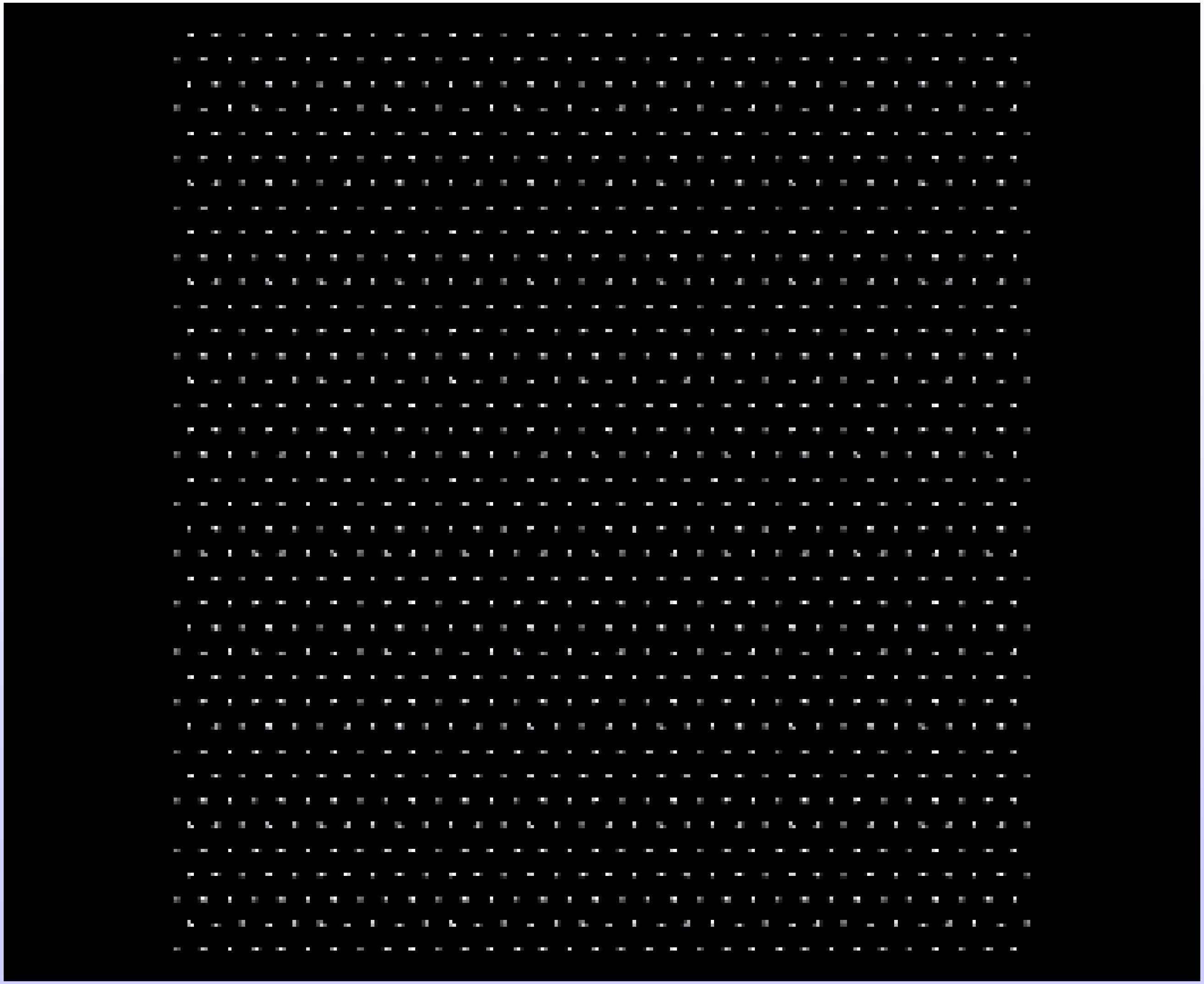
$$\tau_{coll} = \frac{1}{\rho 2 r_c}\sqrt{\frac{m}{\pi T k_B}}$$

infection rate λ

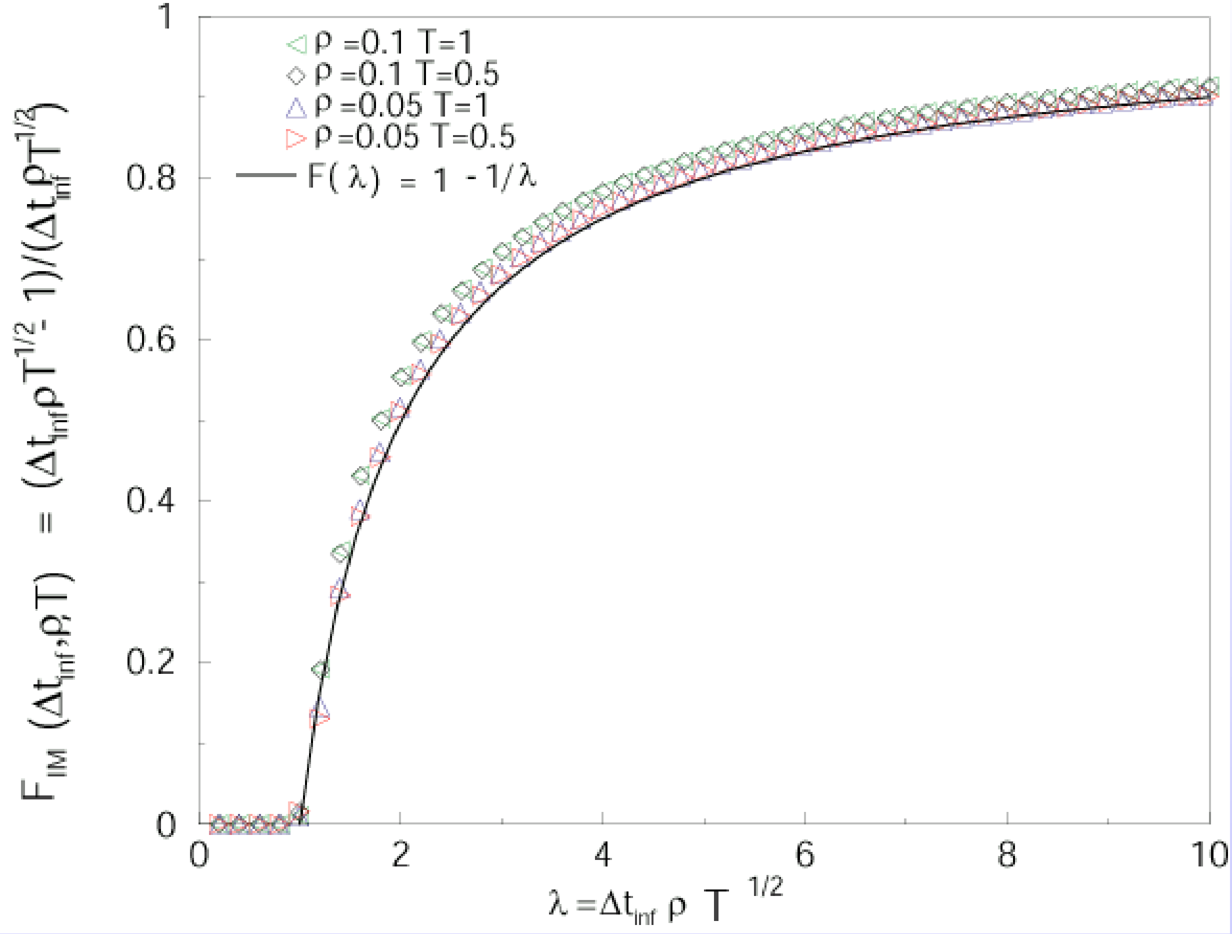
$$\lambda \equiv \frac{\Delta t_{\inf}}{\tau_{coll}} \propto \Delta t_{\inf}\ \rho\ T^{1/2}$$

MC González, PG Lind, HJH,Phys.Rev.Lett. 96, 088702 (2006)

Epidemic Spreading with Mobile Agents

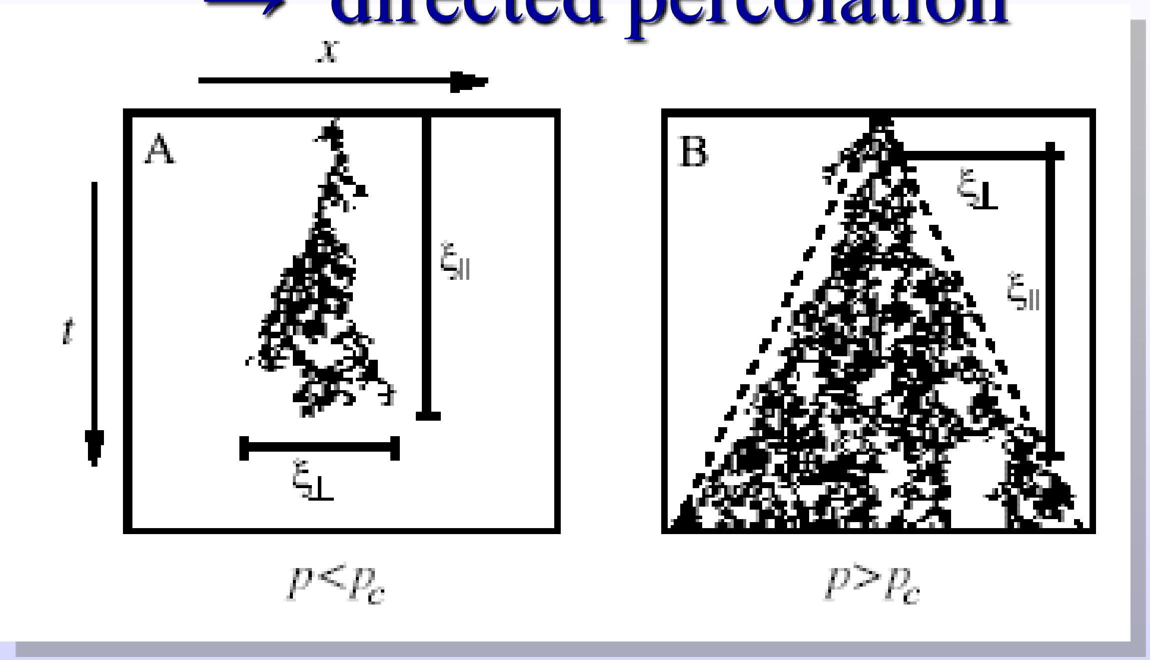


Epidemic Spreading with Mobile Agents



SIS Model on a Lattice

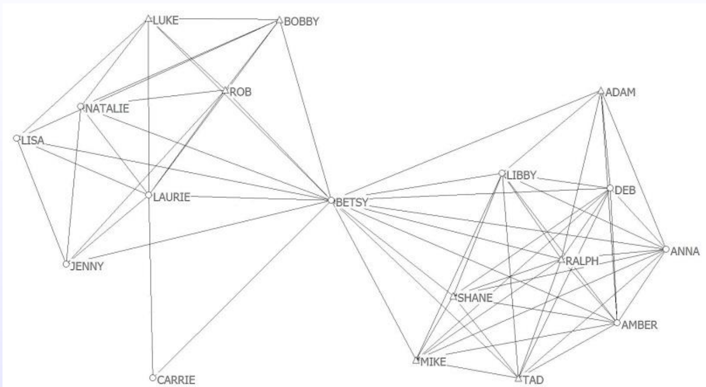
→ directed percolation



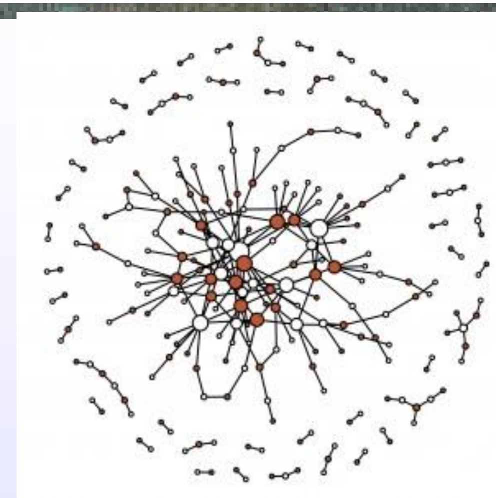
healing phase **spreading phase**

$$i(\infty) \sim (\lambda - \lambda_c)^{\beta}\,,\ \ \beta = 0.586\ldots$$

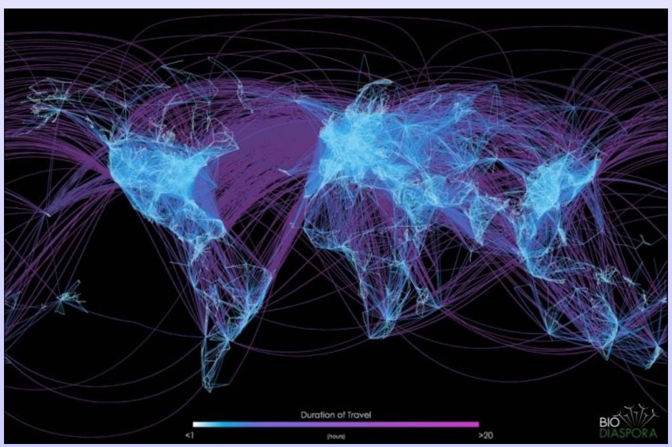
SIS Model on Complex Networks



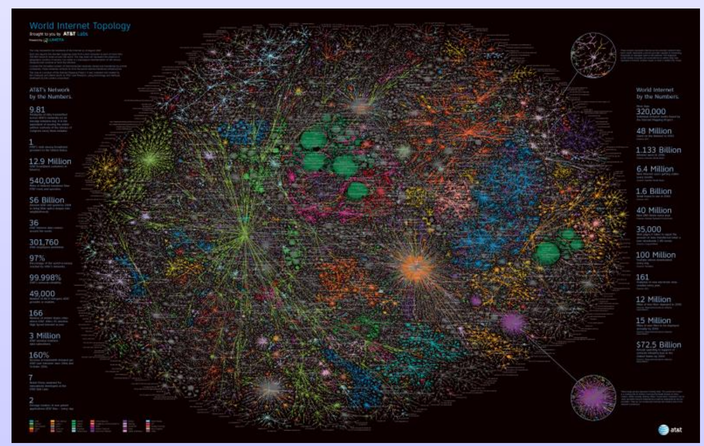
friendship network within a school class



sexual contact network

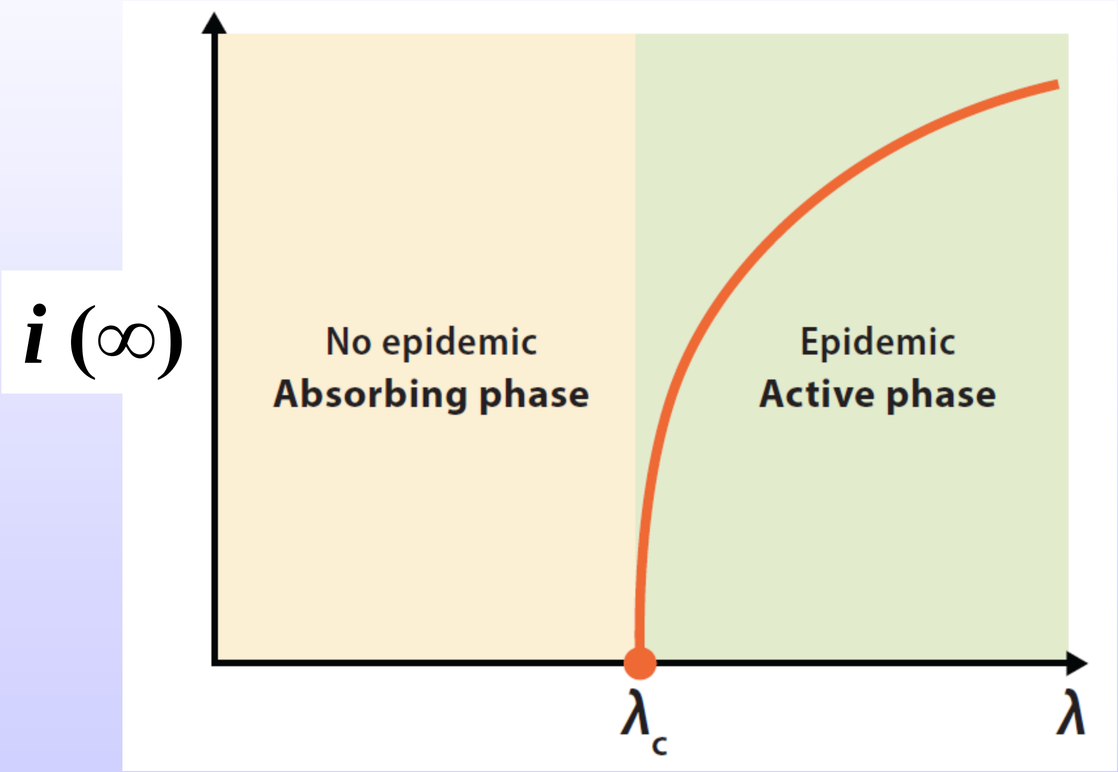


airline network



network of the internet

SIS Model on Complex Networks



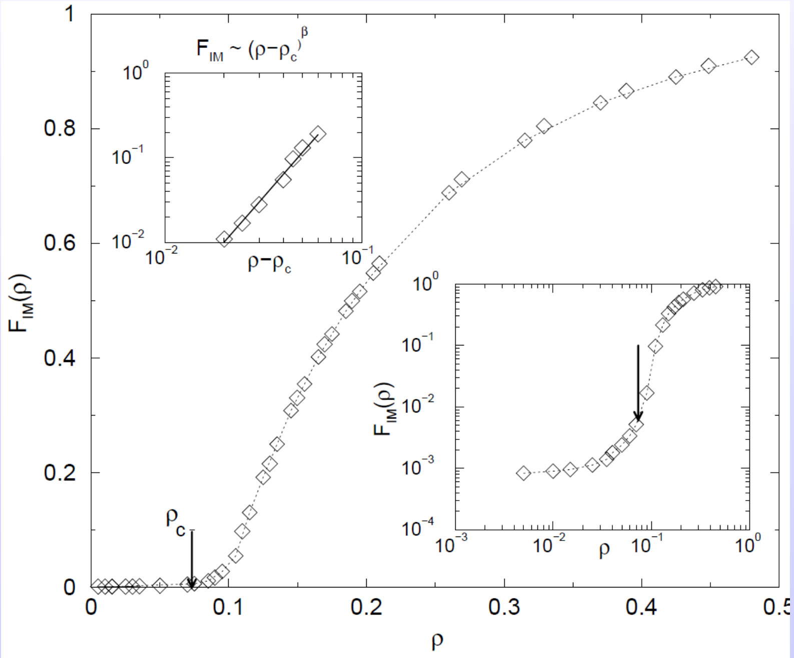
$$i\,(\infty) \sim (\lambda - \lambda_c)^{\beta}$$

$$\beta \in (0,1]$$

Power-law distributed infection times

$$P(\Delta t_{inf}) = (\gamma - 1)\Delta t_{inf}^{-\gamma}$$

$$\Delta t_{inf} \geq 1$$

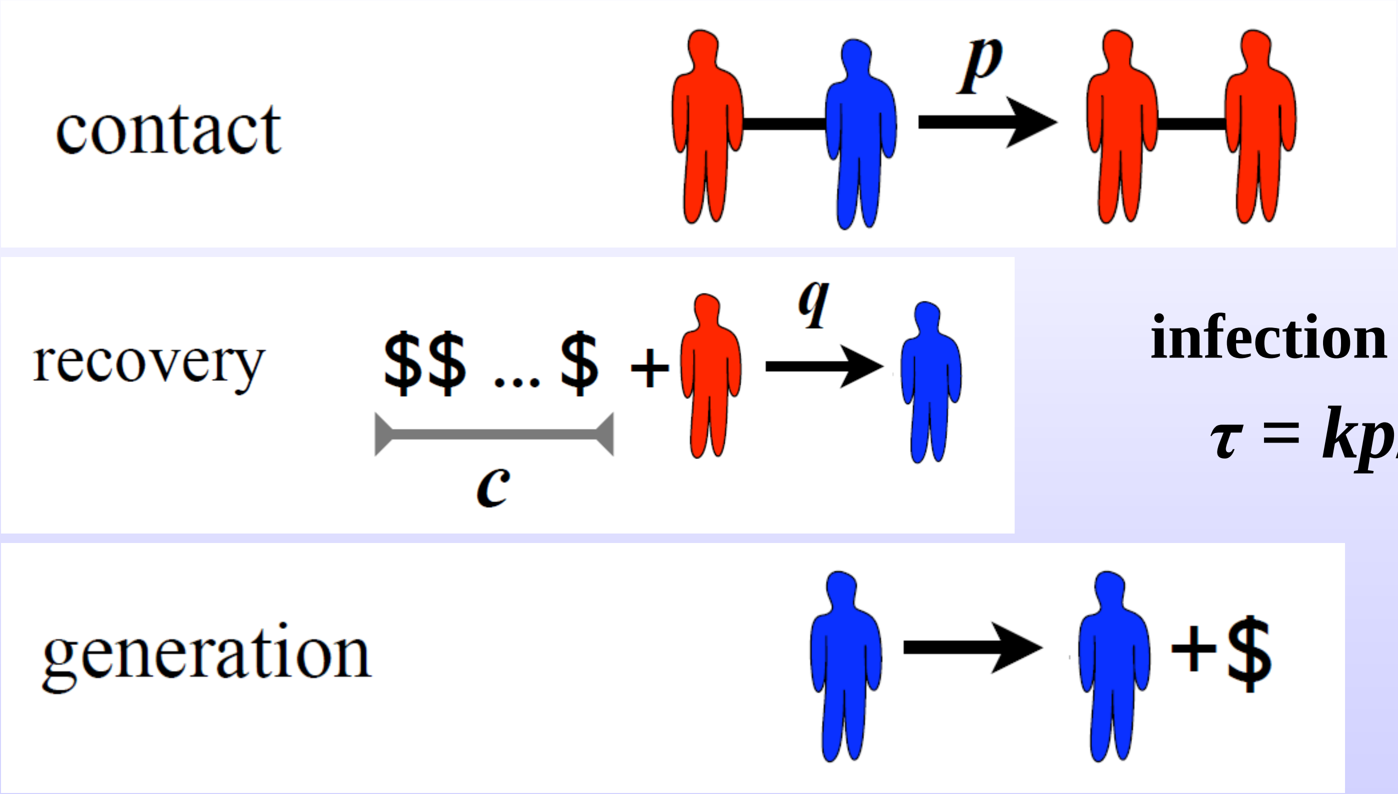


For 2 < γ < 3 the epidemic threshold tends to zero, but exhibits an inflection point at $\lambda_c = 1$.

MC González and HJH, Physica A **340**, 741 (2004)

Epidemics with Global Budget

Budget-constrained Susceptible-Infected-Susceptible (bSIS) model



infection rate

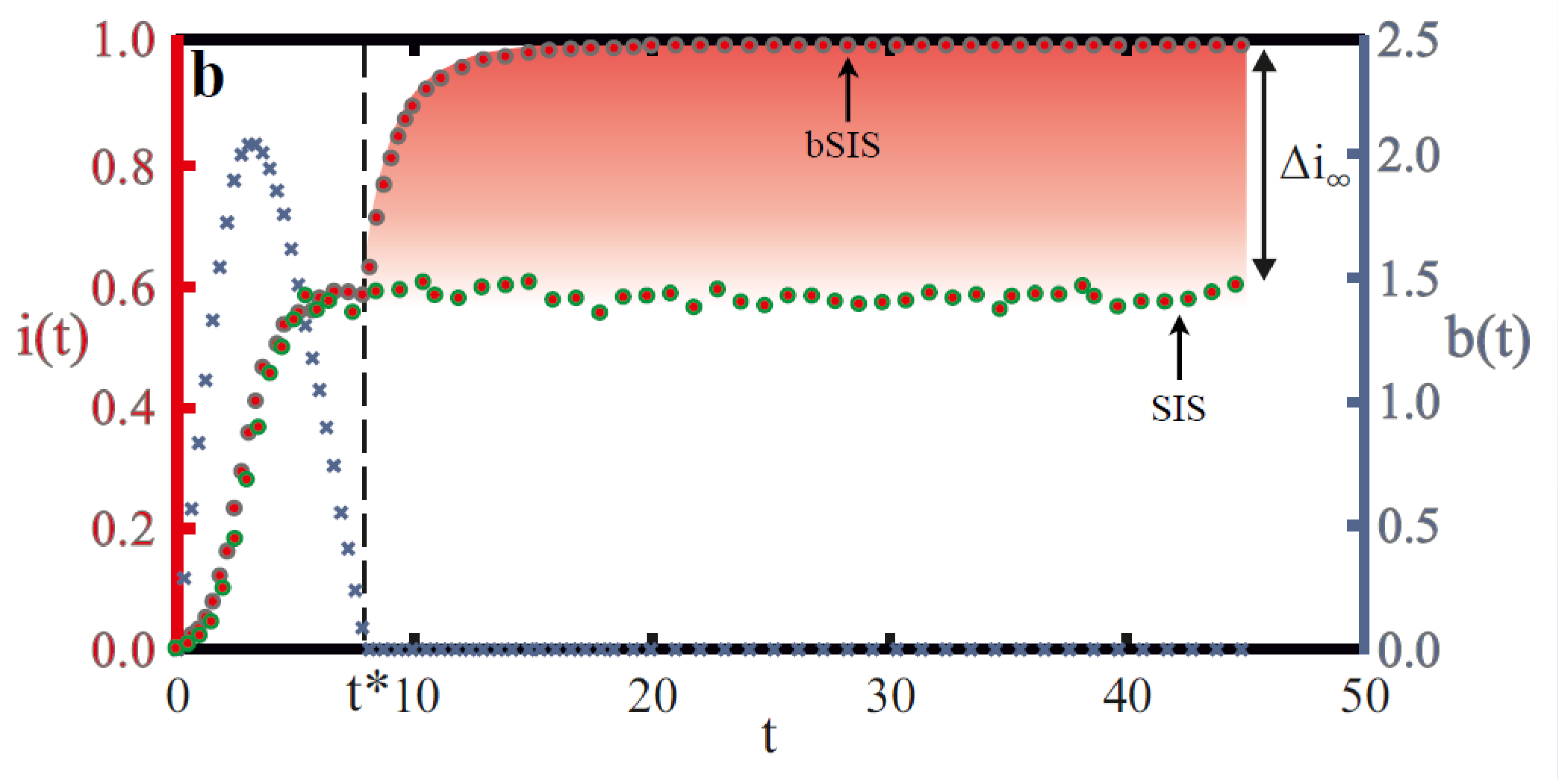
$$\tau = kp/q$$

L. Böttcher, O. Wooley-Meza, N.A.M. Araújo, H.J.H., D. Helbing
Scientific Reports 5, 16571 (2015)

Epidemics with Global Budget

Time evolution in the epidemic regime:

$\tau > \tau^*$



c = 2

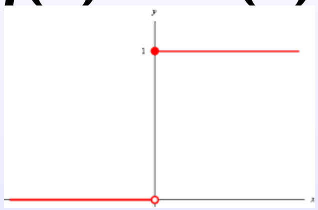
Epidemics with Global Budget

Evolution of budget $b(t)$:

budget function

$$\frac{db(t)}{dt} = s(t) - cqf(b)i(t)$$

$f(b) := \Theta(b)$



susceptible

infected

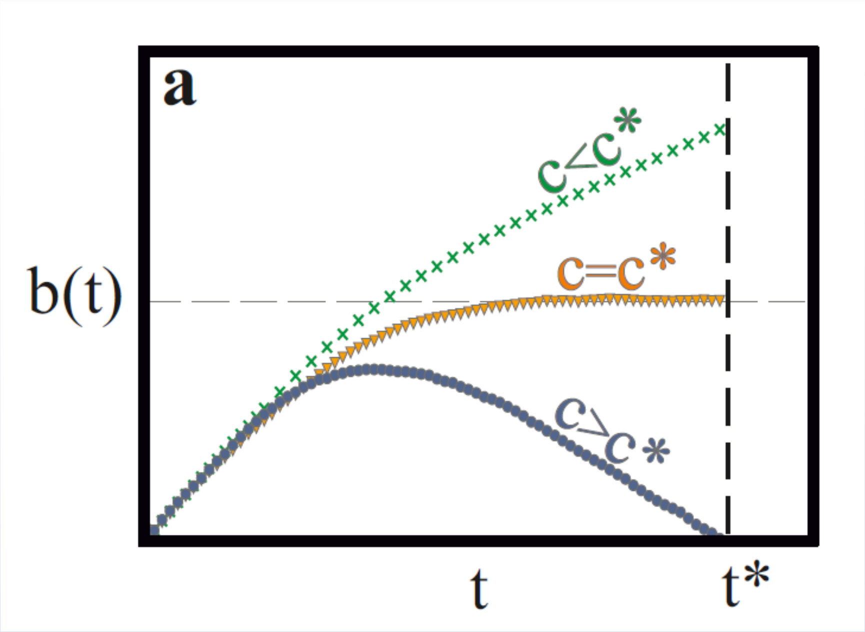
$i(t) = 1 - s(t)$

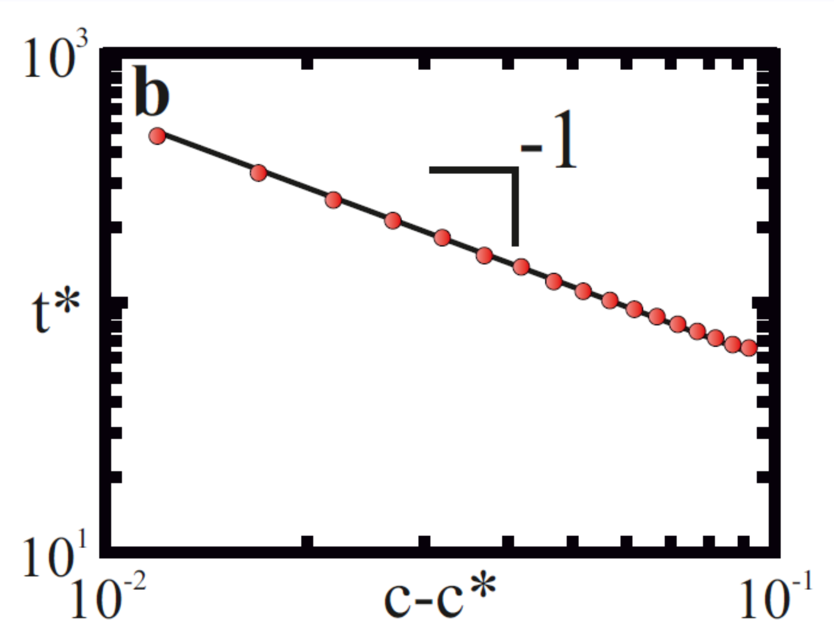
Mean-field:

$$\frac{di(t)}{dt} = kpi(t)s(t) - qf(b)i(t)$$

Effective infection rate: $\tau = kp/q$

Epidemics with Global Budget





time for budget collapse

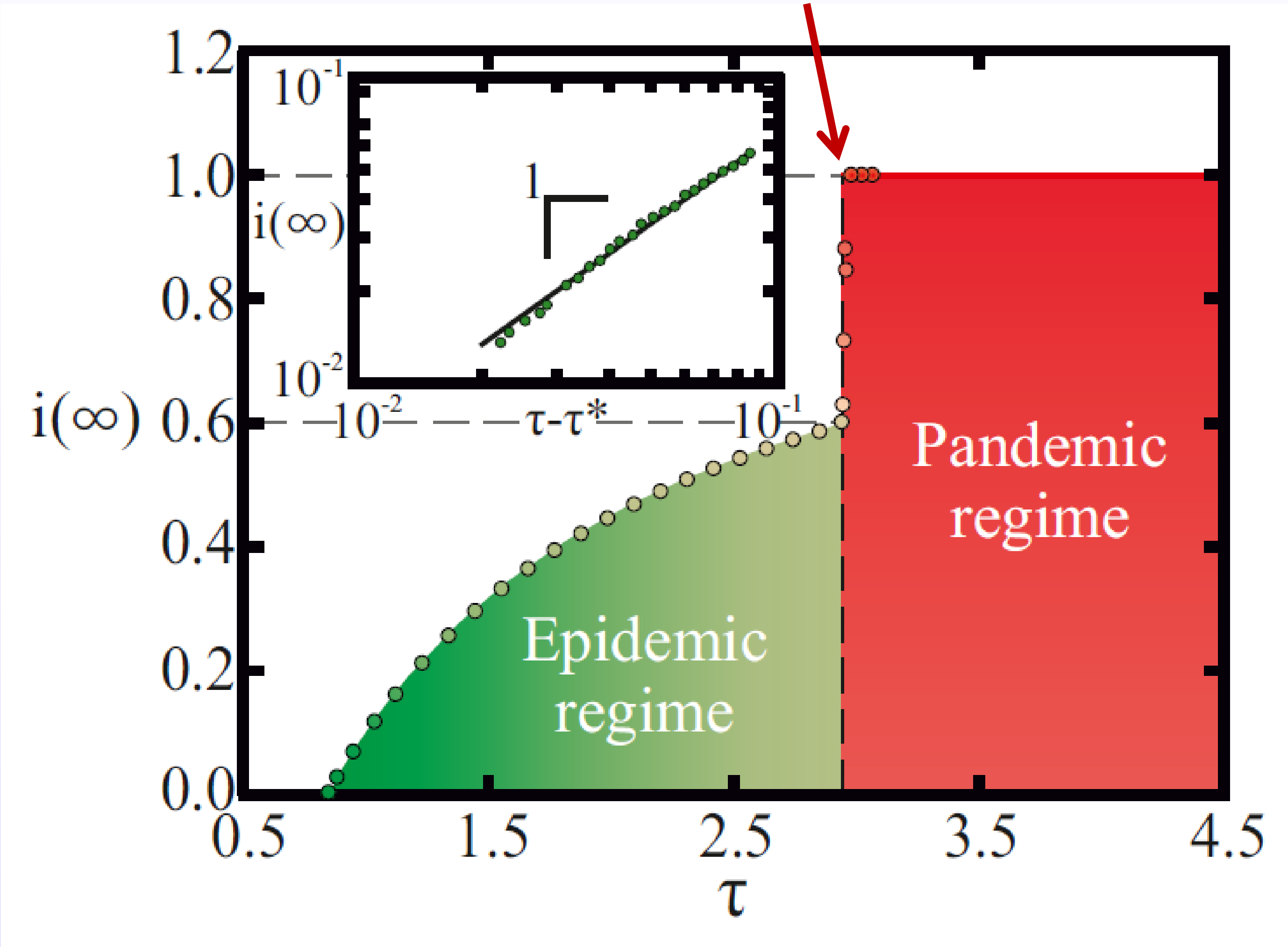
$$t^* \propto \left(c - c^*\right)^{-1}$$

Mean-field: $c^* = \left(kp - q\right)^{-1}$

q = 0.8; p = 0.285

Epidemics with Global Budget

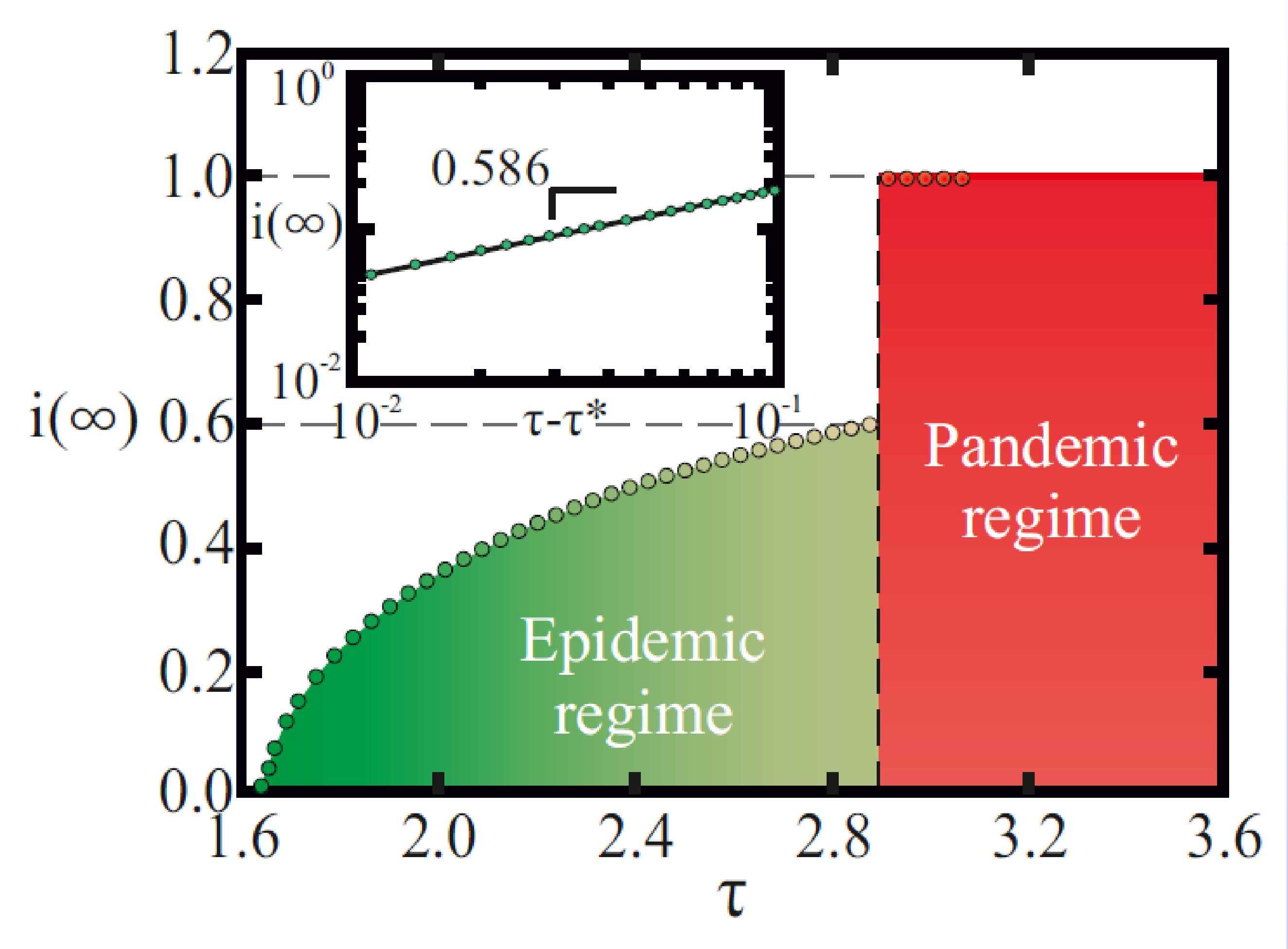
discontinuous transition **(no hysteresis)**



c = 0.833
q = 0.8

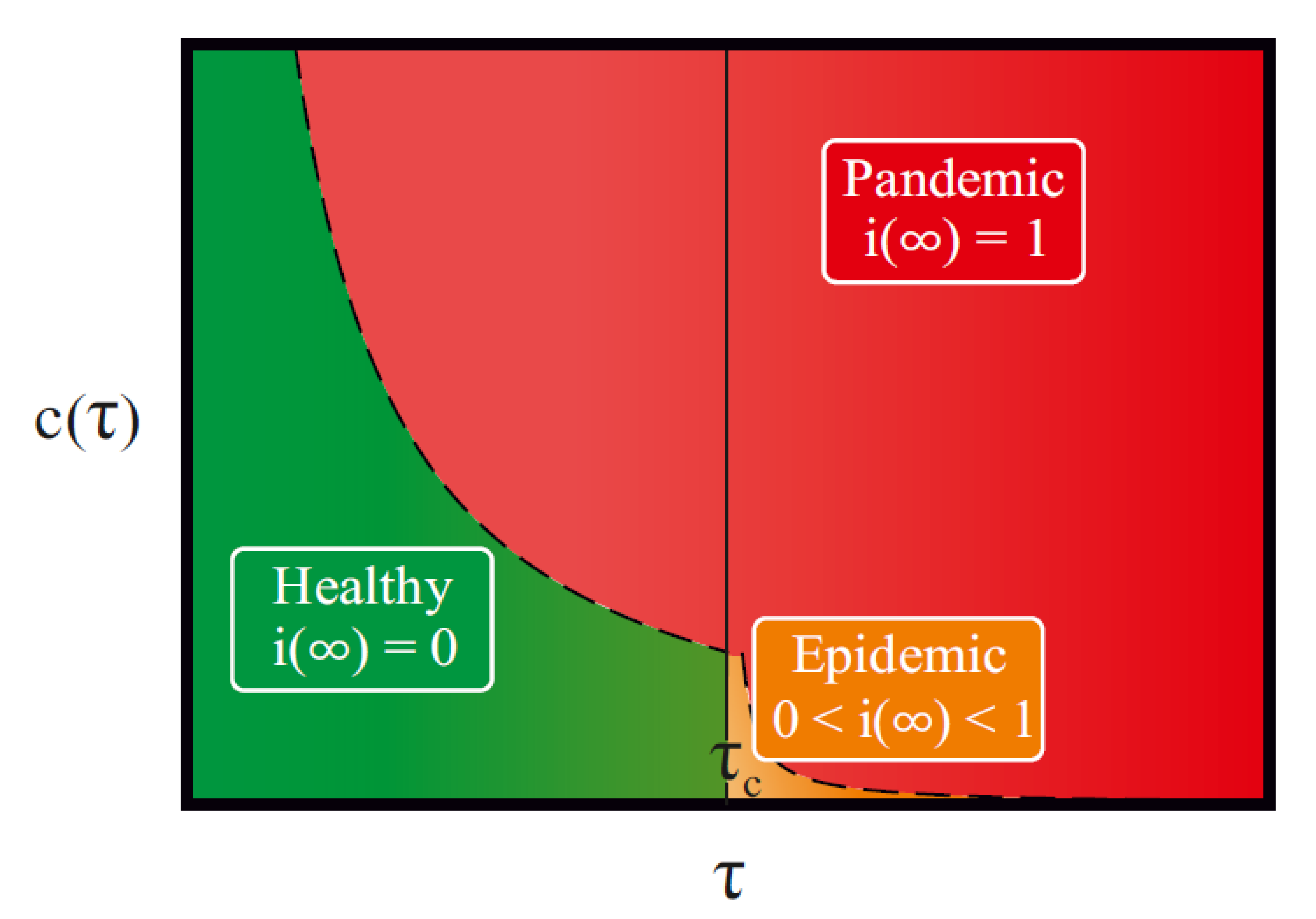
Epidemics with Global Budget

Square lattice



c = 0.833
q = 0.8

Epidemics with Global Budget

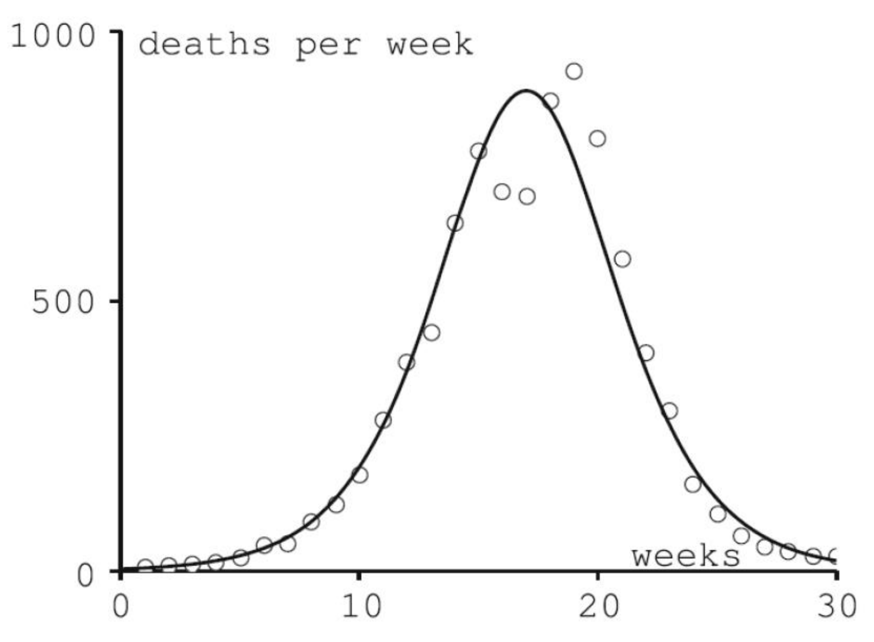


SIR Model

with immunity

$$S \xrightarrow{p} I \xrightarrow{q} R$$

R = recovered = resistant



outbreak of the plague in Bombay between 1905 and 1906

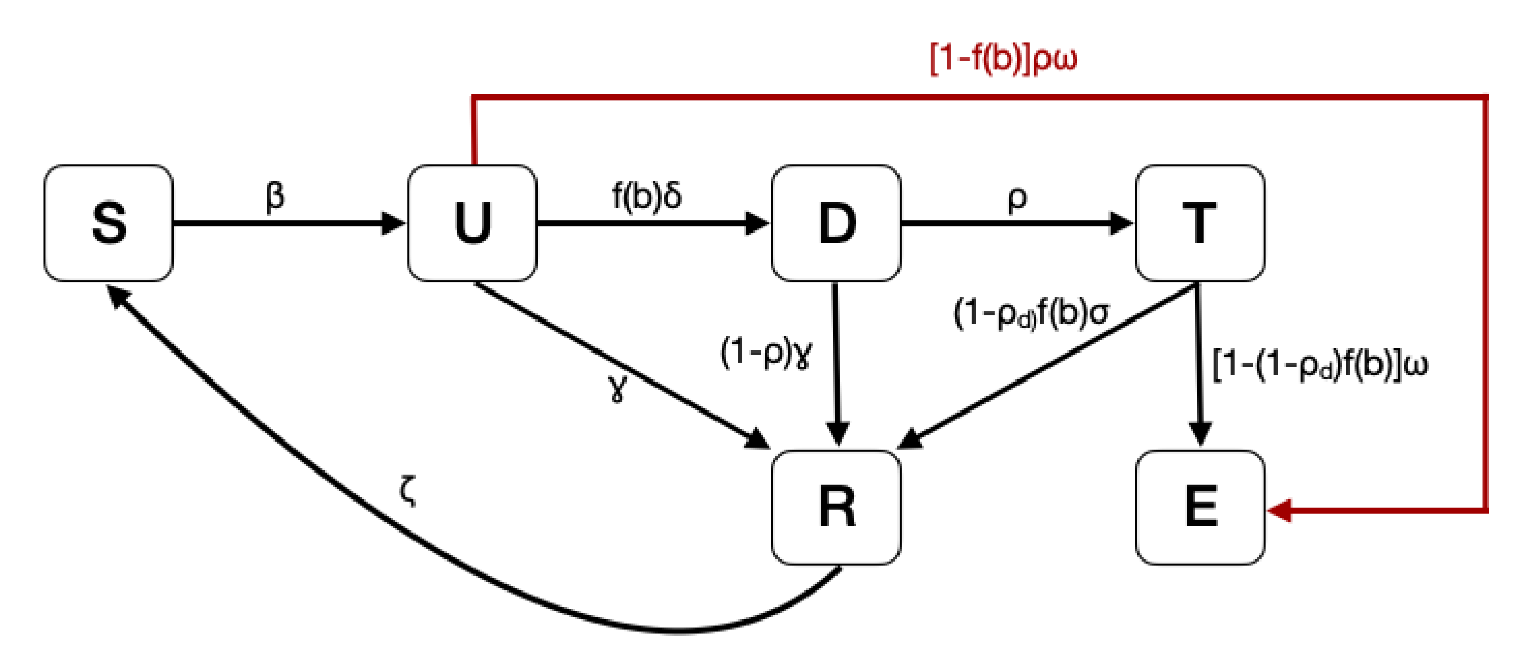
$$\frac{ds}{dt} = -\, kp\, s(\mathrm{t})\, i(\mathrm{t})$$

$$\frac{di}{dt} = -\, q\, i(\mathrm{t}) + kp\, s(\mathrm{t})\, i(\mathrm{t})$$

$$\frac{dr}{dt} = q\, i(\mathrm{t})$$

$$s(\mathrm{t}) + i(\mathrm{t}) + r(\mathrm{t}) = 1$$

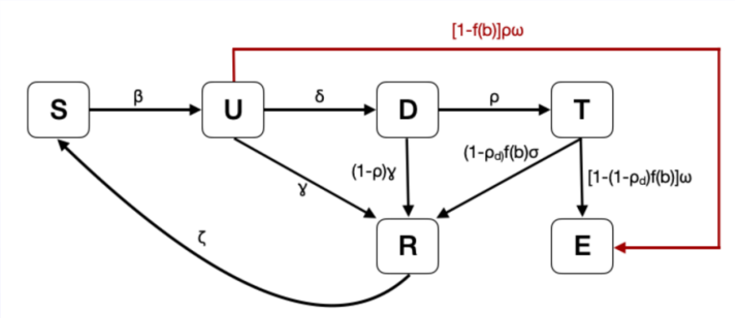
Modeling Covid-19



- S : susceptible individuals
- U : undetected ill individuals
- D : detected ill individuals
- R : resistent (immune) individuals
- T : individuals who need treatment
- E : for extinct individuals

M. Aucouturier and H.J.H, Int. J. Mod. Phys. C in press

Modeling Covid-19



b = budget

$f(b)$ is Heaviside function

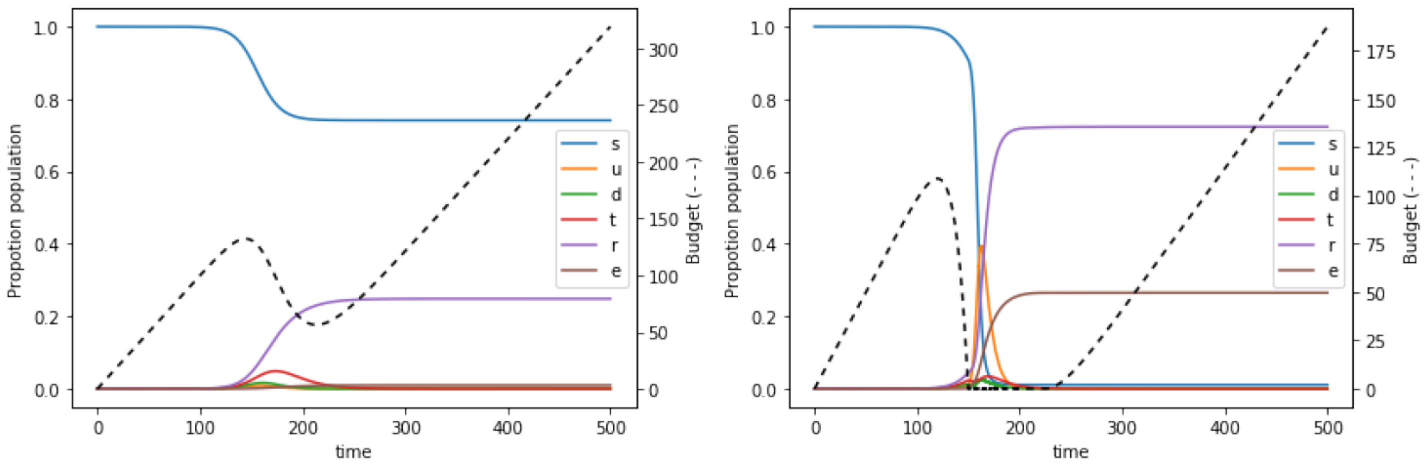
$$\begin{aligned}
\dot{s} &= -\beta.s.u + \zeta.r \\
\dot{u} &= \beta.s.u - \delta.f(b).u - \gamma.u - [1 - f(b)].\rho.\omega.u \\
\dot{d} &= \delta.f(b).u - (1-\rho).\gamma.d - \rho.d \\
\dot{t} &= \rho.d - (1-\rho_d).\sigma.f(b).t - [1 - (1-\rho_d).f(b)].\omega.t \\
\dot{r} &= \gamma.u + (1-\rho).\gamma.d + (1-\rho_d).\sigma.f(b).t - \zeta.r \\
\dot{b} &= s + r - c_t.(1-\rho_d).f(b).t
\end{aligned}$$

$$s + u + d + t + r + e = 1$$

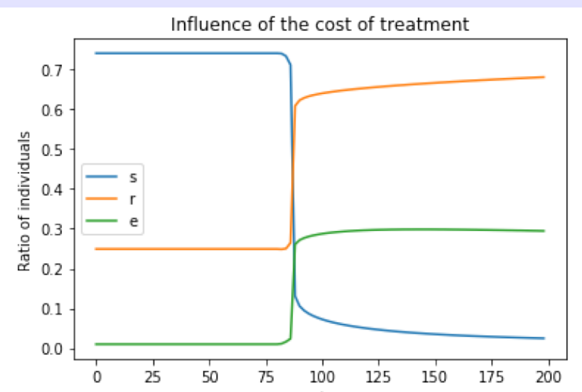
Modeling Covid-19

Parameter	Value	Meaning
β	$\ln(2)$	Infection rate
δ	0.5	Testing rate
γ	$\ln(2)/7$	Natural recovery rate
σ	$1/20$	Recovery rate of individuals in treatment
ω	$1/9$	Death rate of individuals in treatment
ζ	0	Rate of the loss of immunity
ρ	0.2	Fraction of individuals needing treatment
ρ_d	0.03	Proportion of inevitable deaths
c_t	60	Cost of treatment
c_k	0.5	Cost of a test

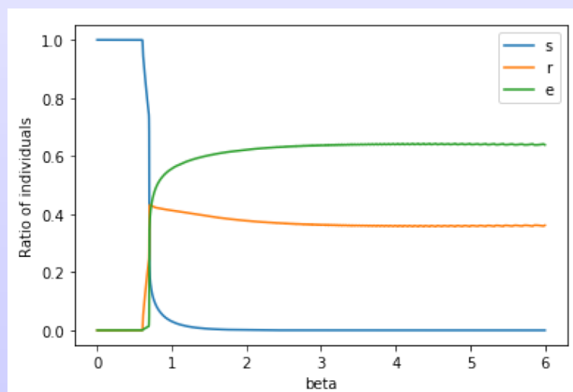
Modeling Covid-19



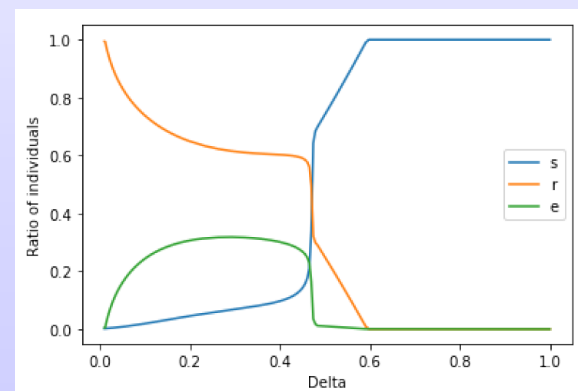
(a) $c_t = 60$ (b) $c_t = 500$



cost of treatment



infection rate

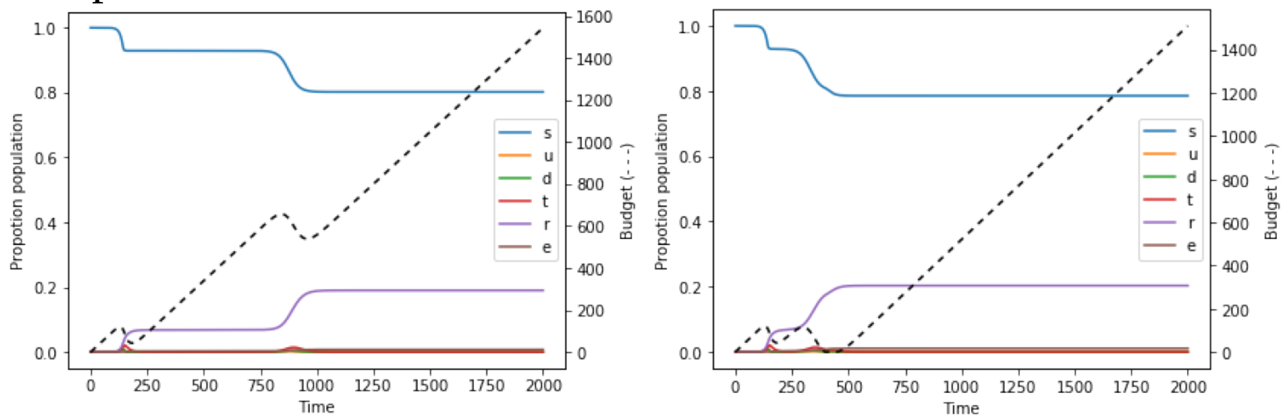


testing rate

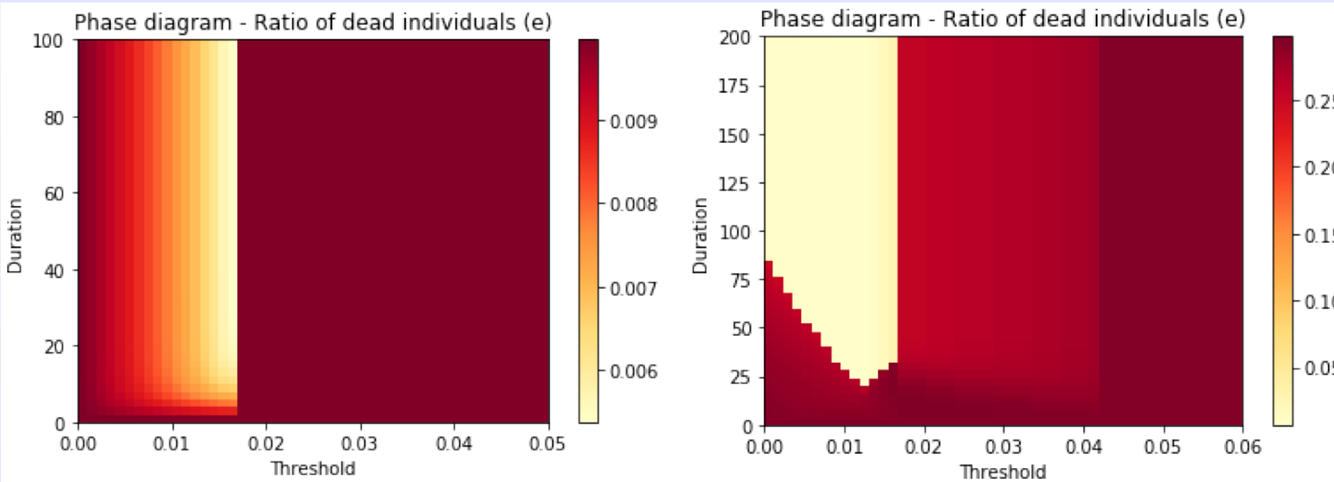
Modeling efficiency of lockdown

without budget collapse

with budget collapse



(a) τ=100 (b) τ=25

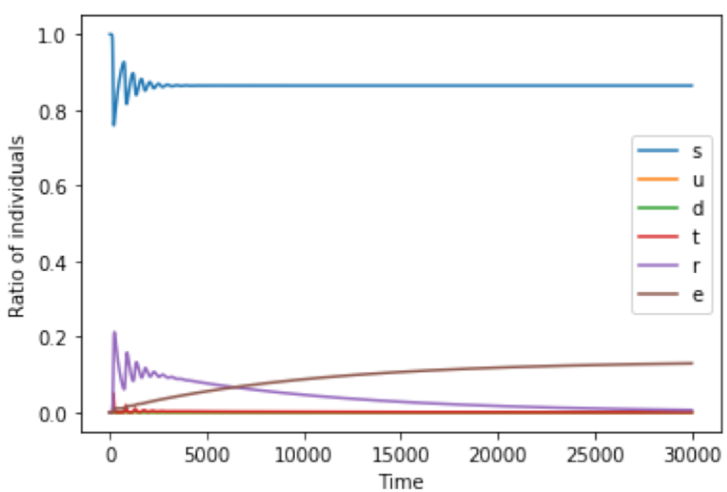


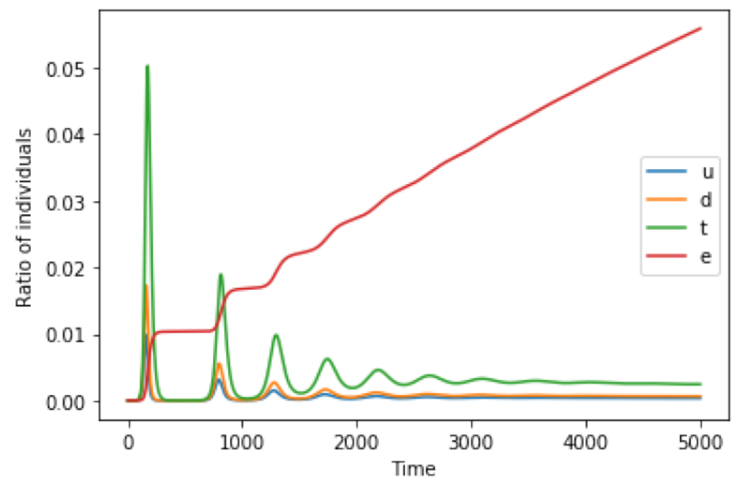
(a) $c_t = 60$ (without budget collapse) (b) $c_t = 200$ (with budget collapse)

threshold = fraction of detected people at which one starts lockdown

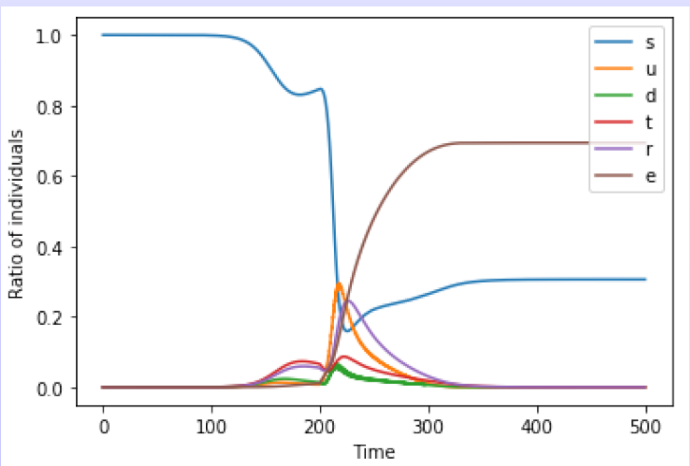
Modeling waning immunity

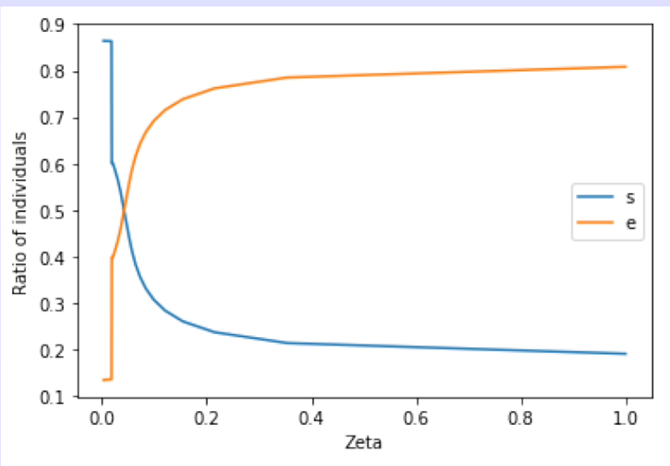
without budget collapse



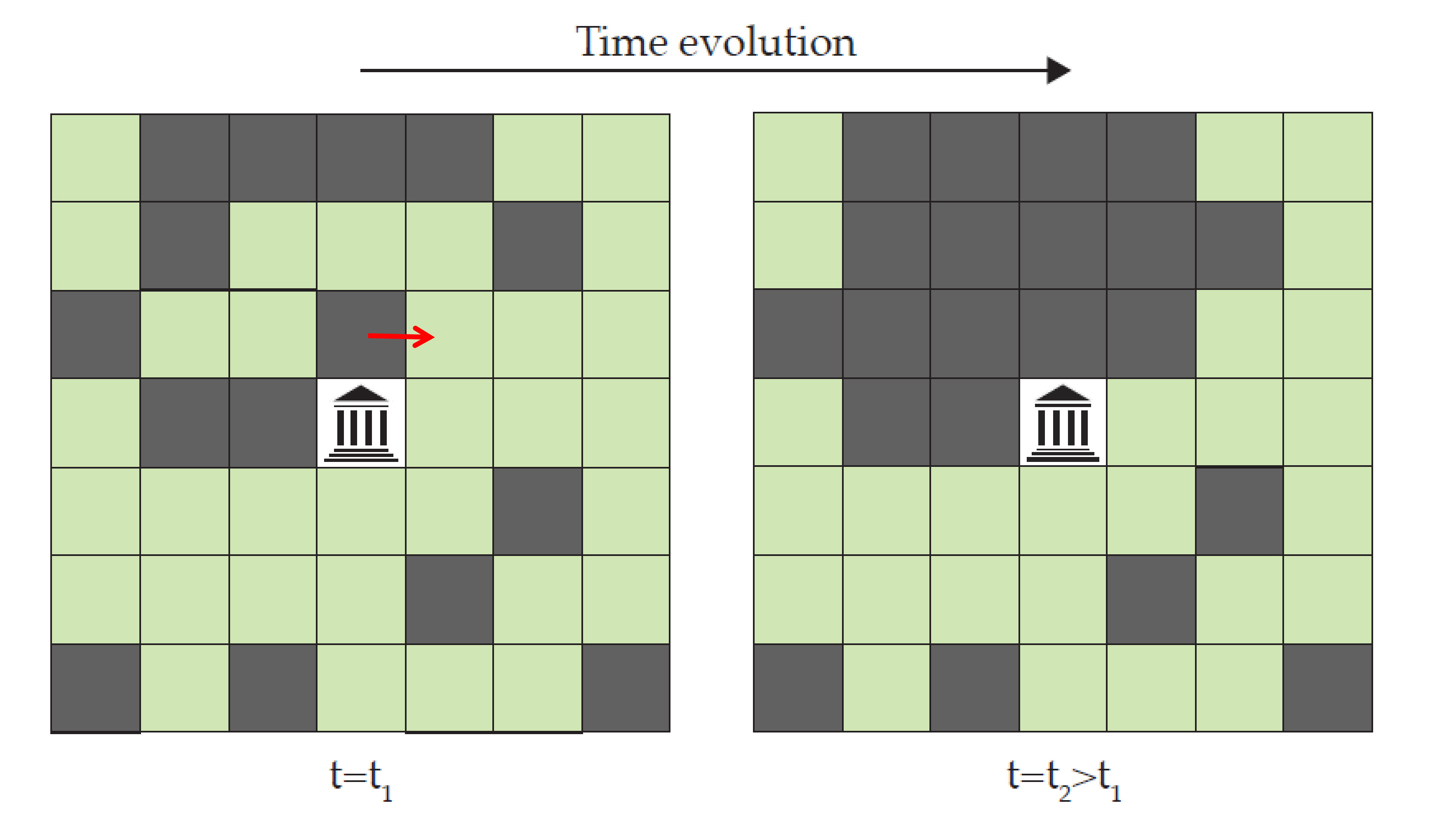


with budget collapse





Requiring a connecting path to the supply center

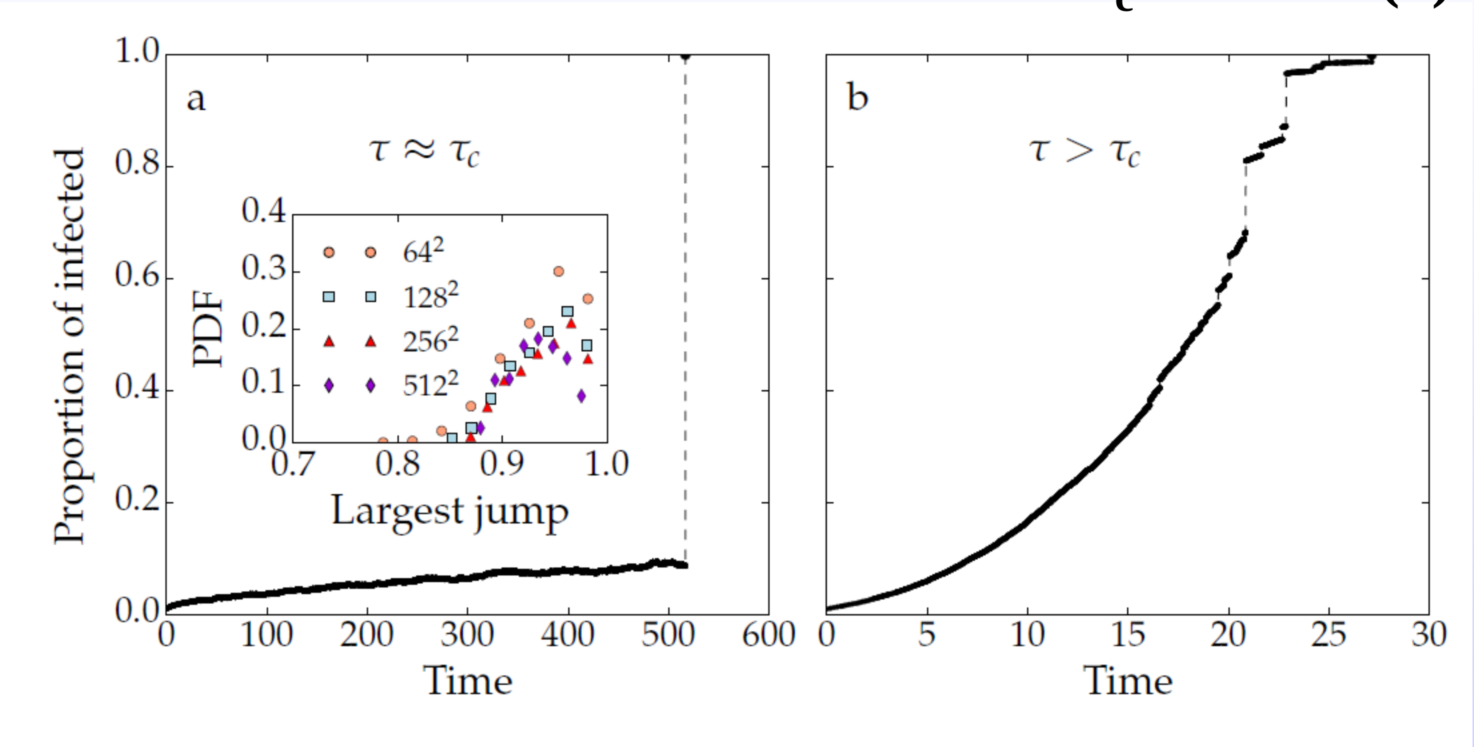


L. Böttcher, O. Wooley-Meza, E. Goles, D. Helbing, H.J.H., Phys. Rev. E 93, 042315 (2016)

Imposing the connecting path to a supply center

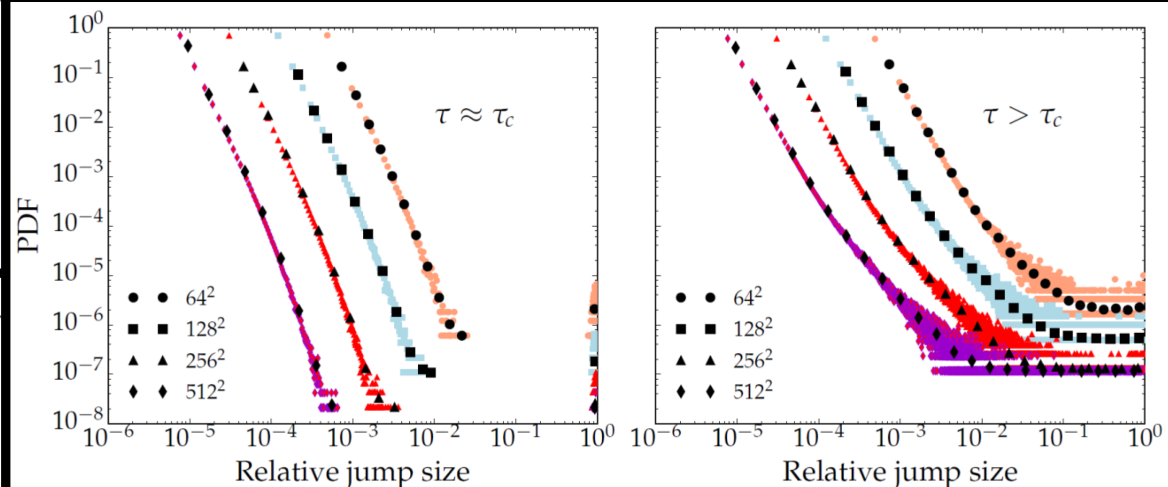
128 × 128 square lattice; q = 0.4

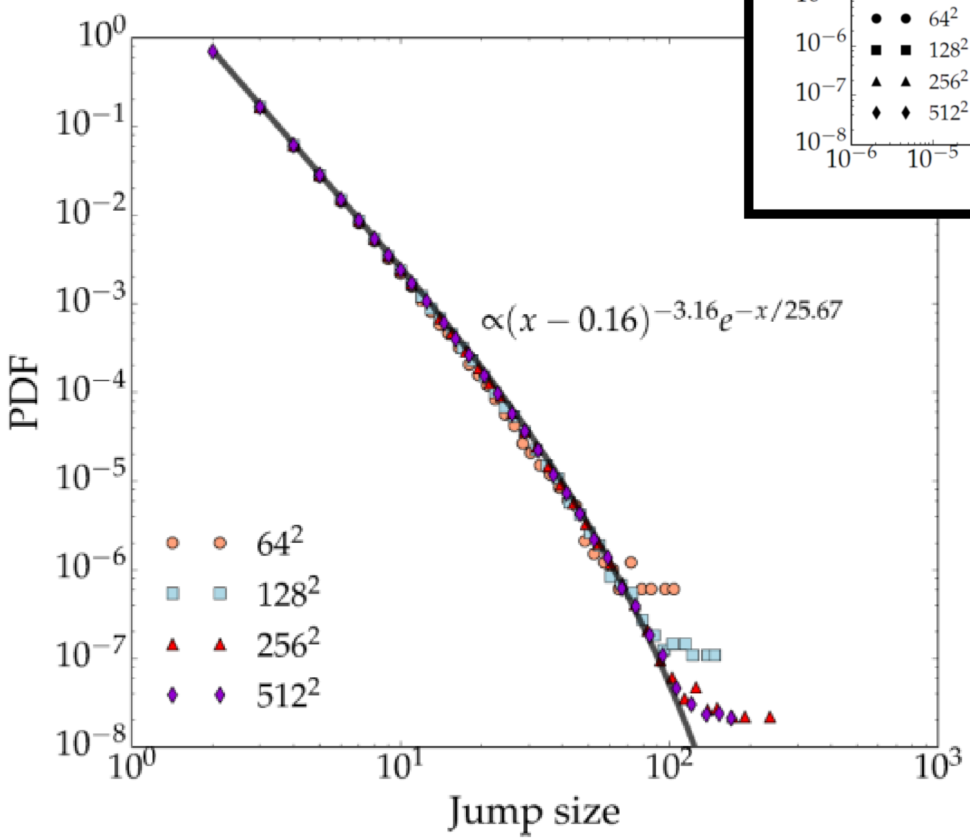
$\tau_c = 1.6488(1)$



p = 0.165 **p = 0.3**

Jump size distribution



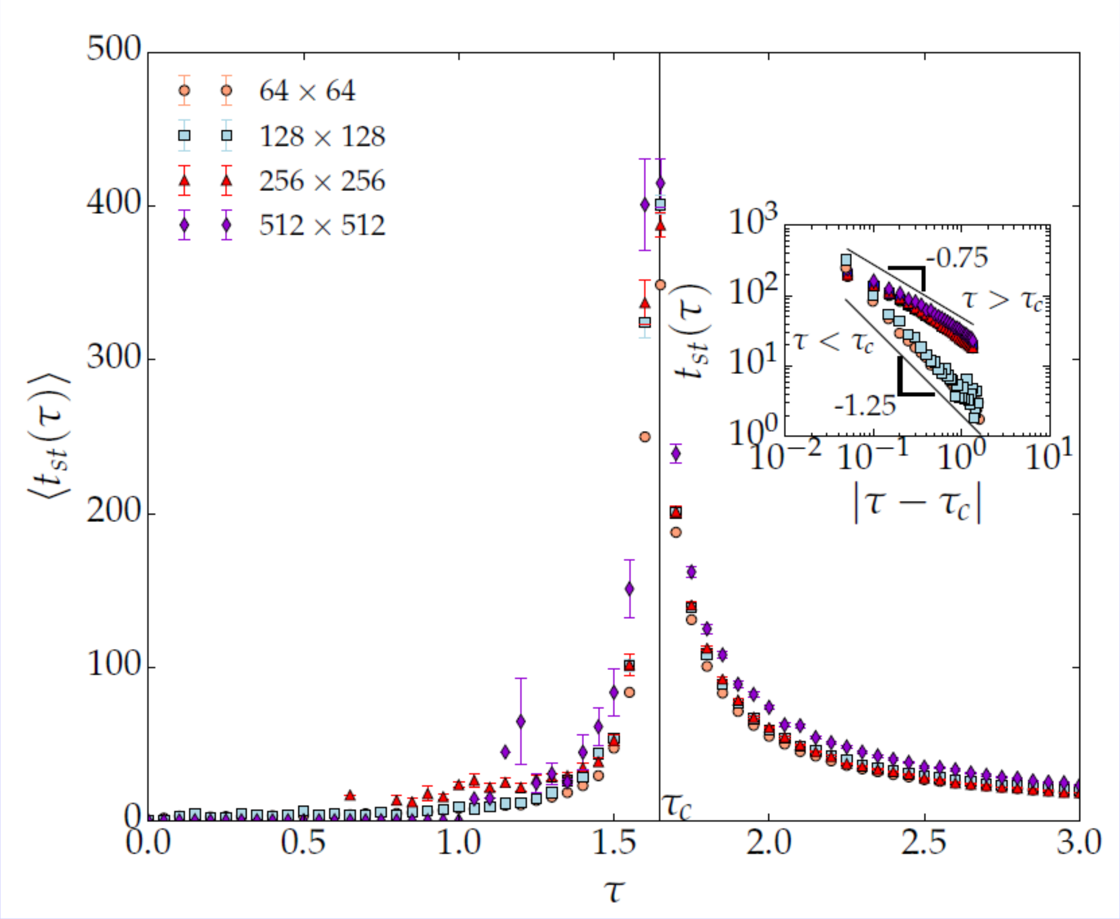


finite size scaling

not a clear power-law

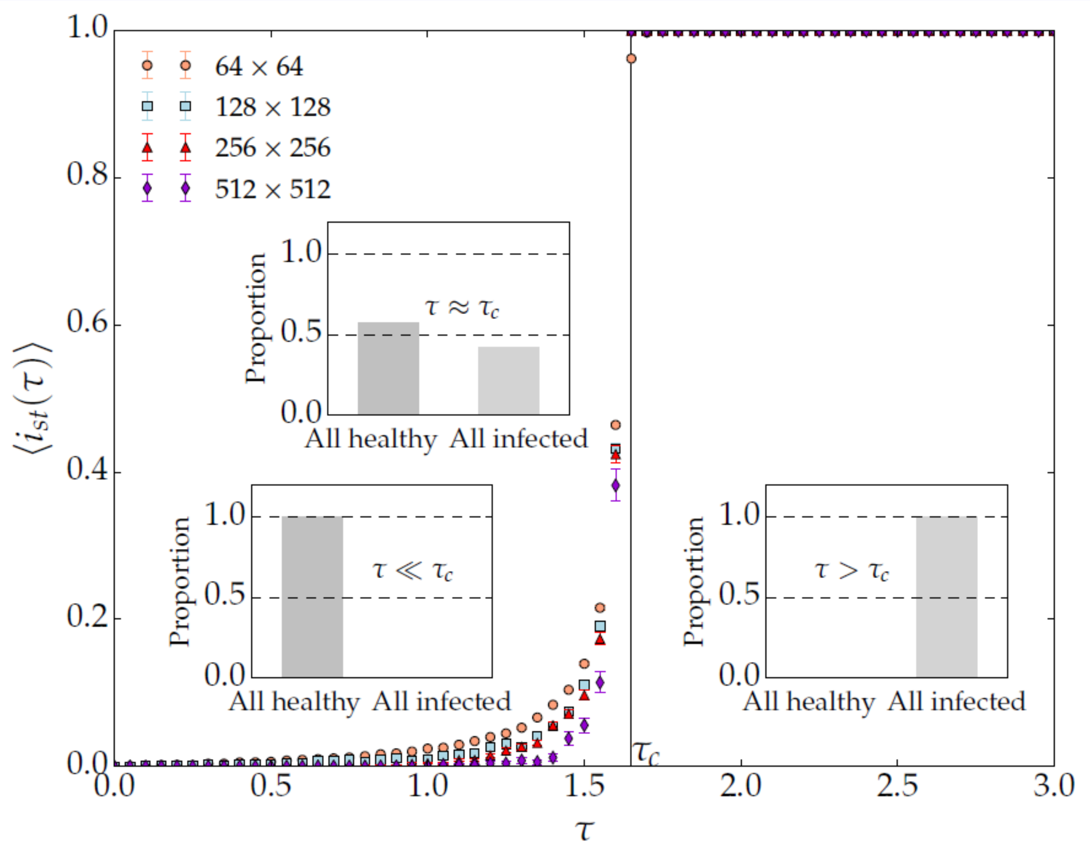
Transition time to fully infected state

square lattice; q = 0.4

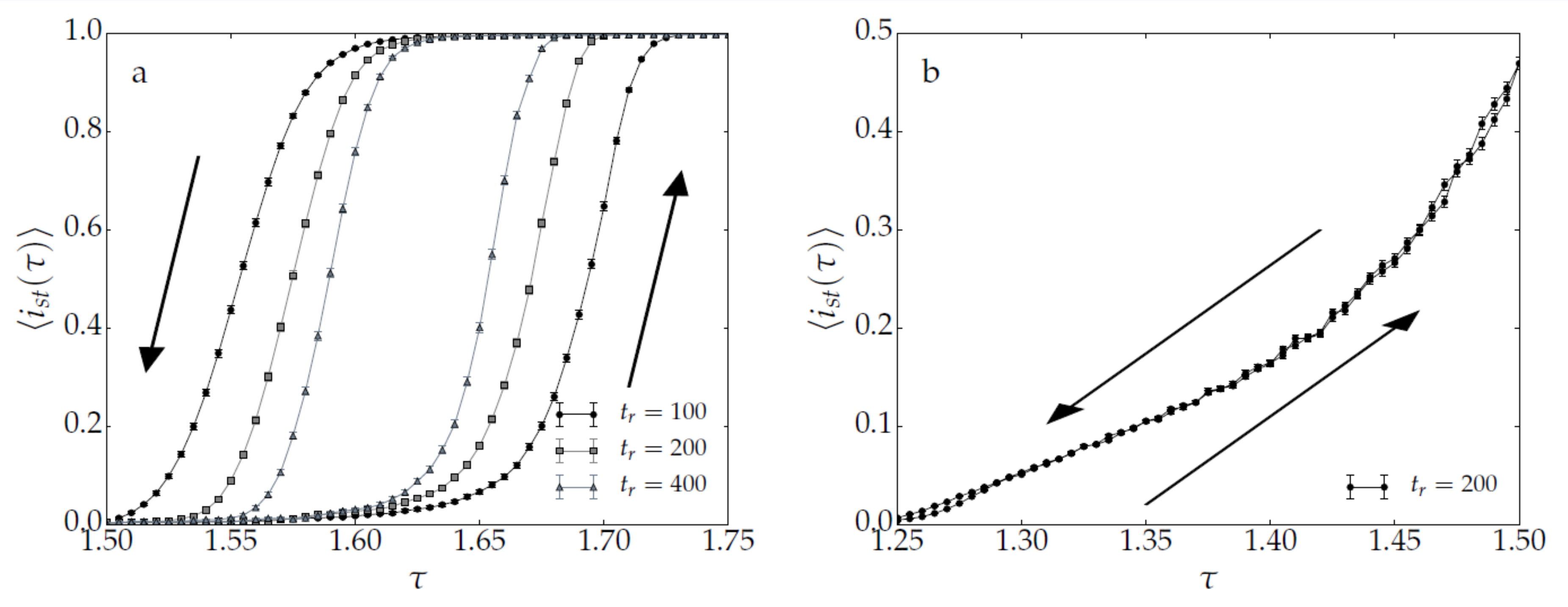


Order Parameter = i(t = ∞)

square lattice; q = 0.4



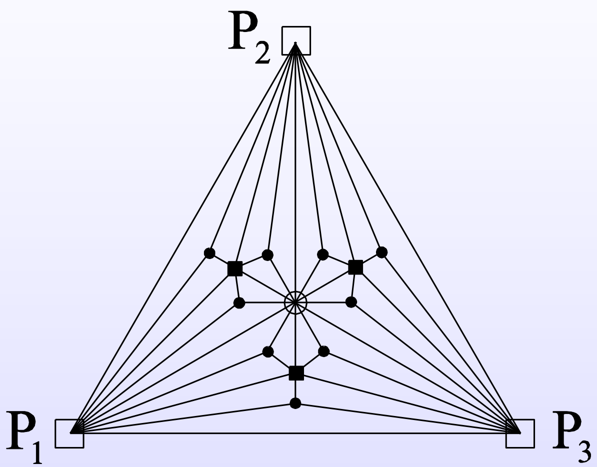
Hysteresis



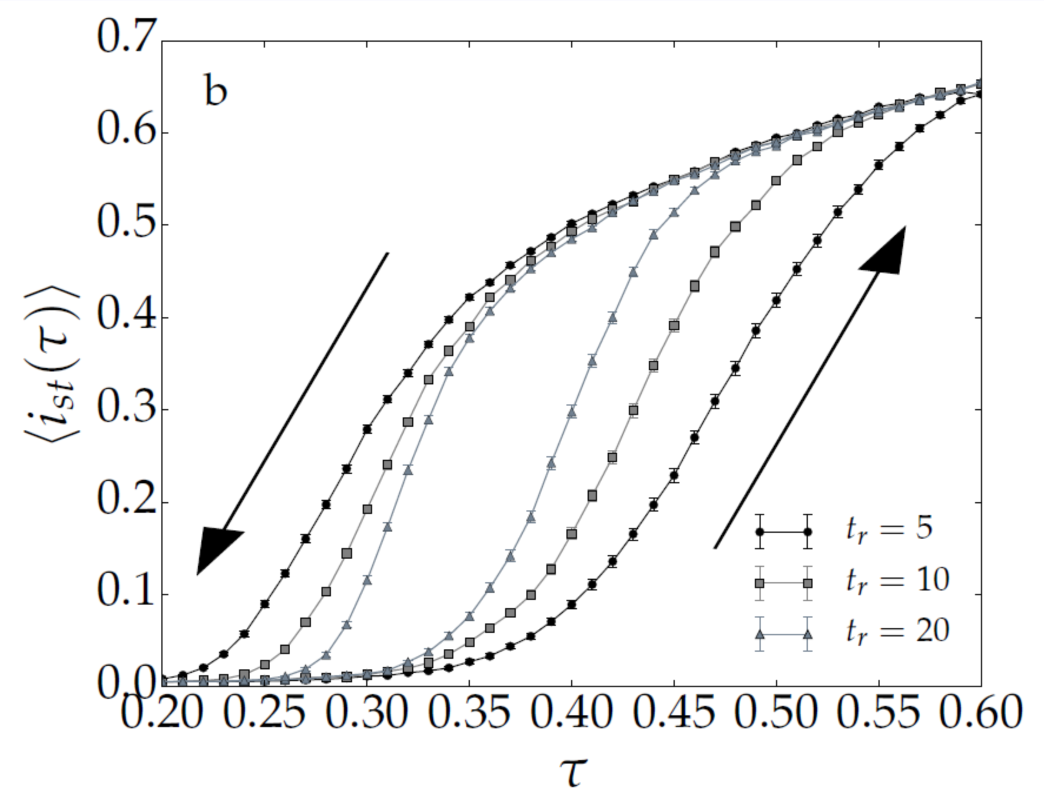
square lattice
for different waiting times t_f

with long range connections
r =0.5; <k> = 4.99

Apollonian network



1096 nodes
<k> = 5.99

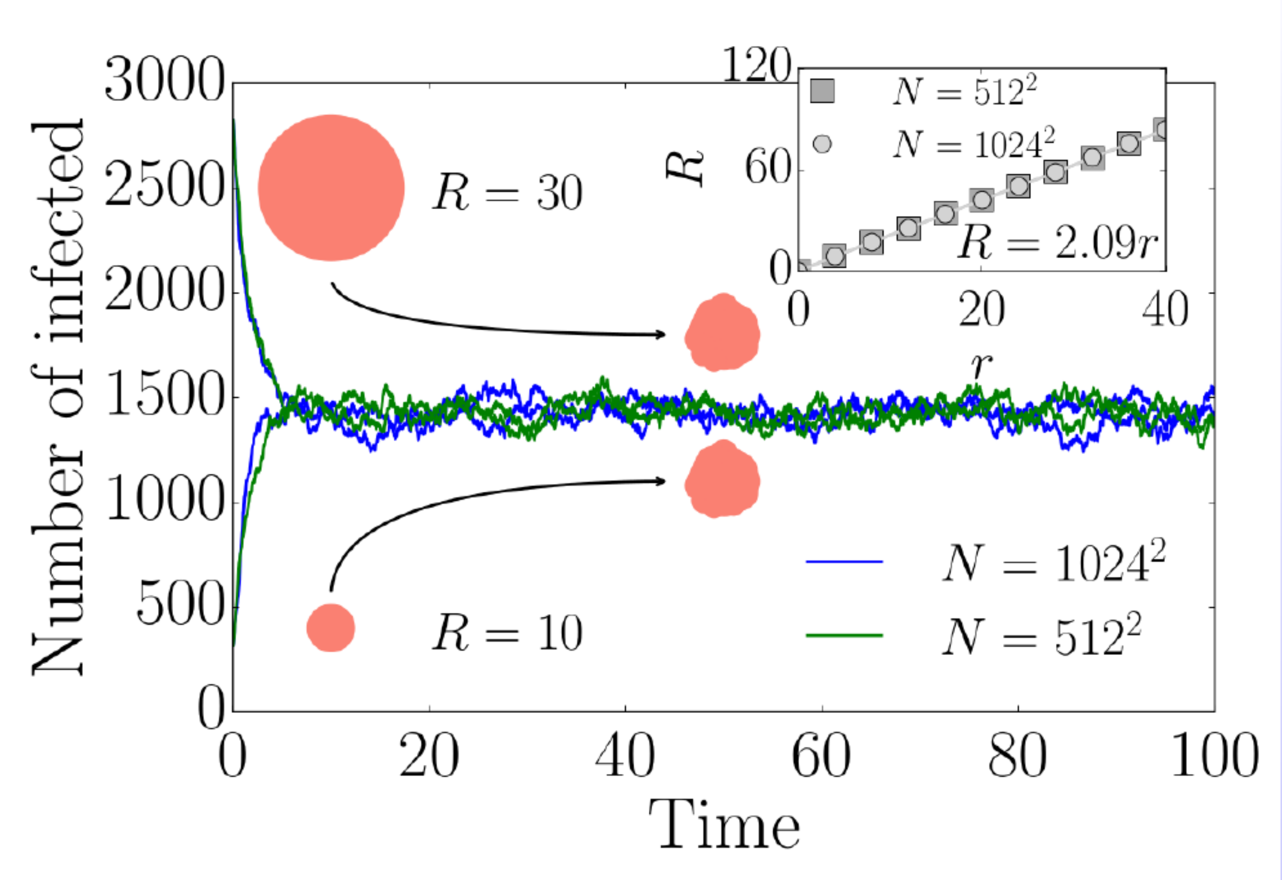


Targetted Recovery

Sites with least number of infected neighbors recover first.

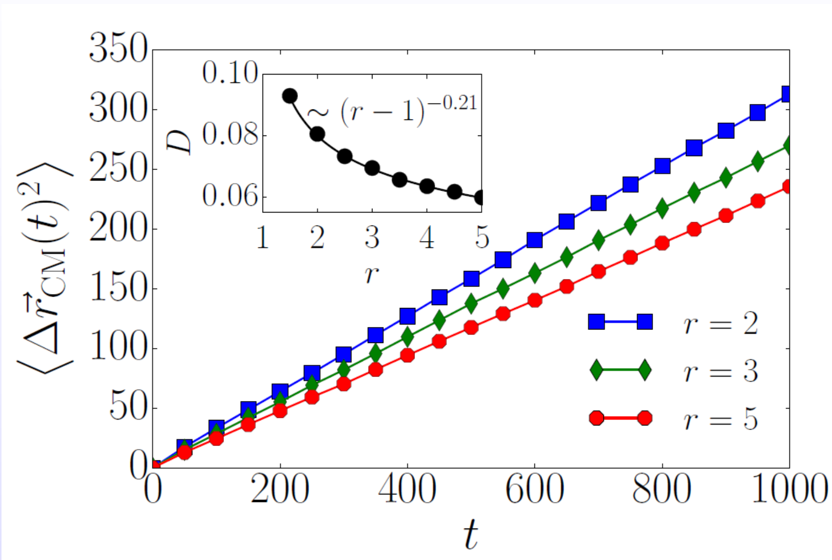
infection rate

r = 10

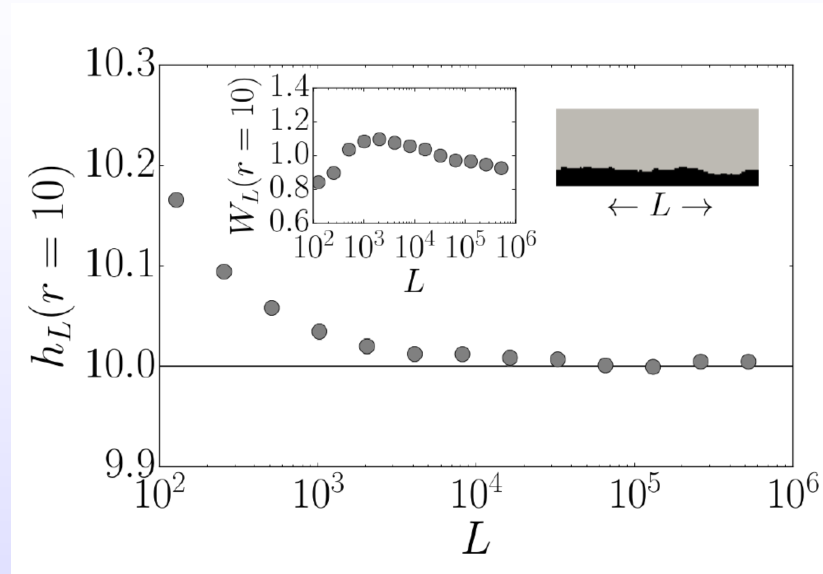


L. Böttcher, J.S. Andrade Jr., H.J.H., Sci. Rep. **7**, 14356 (2017)

Targetted Recovery



diffusion



surface

Metastability in Recovery

Model definition

(i) a node spontaneously fails in a time interval dt with probability pdt (internal failure)
(ii) if fewer than or equal to m nearest neighbors of a certain node are active, this node fails due to external causes with probability rdt (external failure)
(iii) spontaneous recovery with probability qdt (internal recovery) or probability q'dt (external recovery)

L. Böttcher, M. Lukovic, J. Nagler, S. Havlin and H.J.H., Sci. Rep. 7, 41729 (2017)

Mean Field Calculation

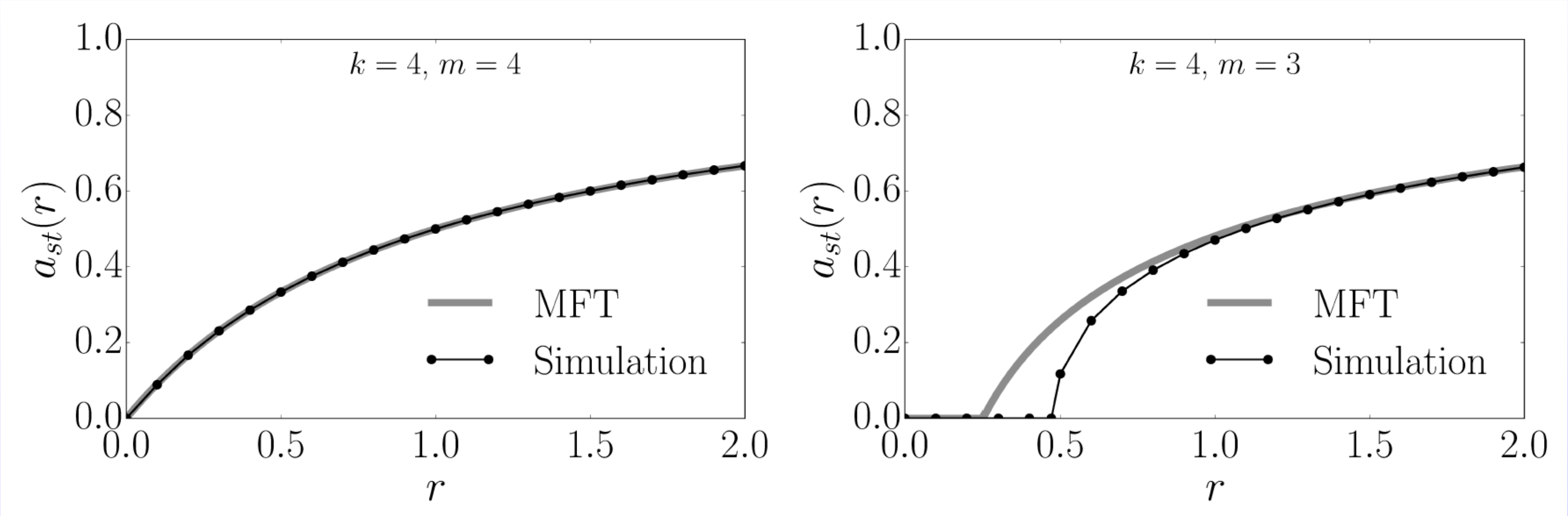
$$\mathrm{i(t)} = \mathrm{u_{int}(t)} + \mathrm{u_{ext}(t)}$$

$$\frac{\mathrm{du_{int}}}{\mathrm{dt}} = \mathrm{p}\,(1 - \mathrm{i(t)}) - \mathrm{q}\ \mathrm{u_{int}}\,(\mathrm{t})$$

$$\frac{\mathrm{du_{ext}}}{\mathrm{dt}} = \mathrm{r}\sum_k f_k E_k\,(1 - \mathrm{i(t)}) - \mathrm{q'}\ \mathrm{u_{ext}}\,(\mathrm{t})$$

$$E_k = \sum_{j=0}^{m} \binom{k}{k-j} (i(t))^{k-j} (1 - i(t))^{j}$$

Order parameter

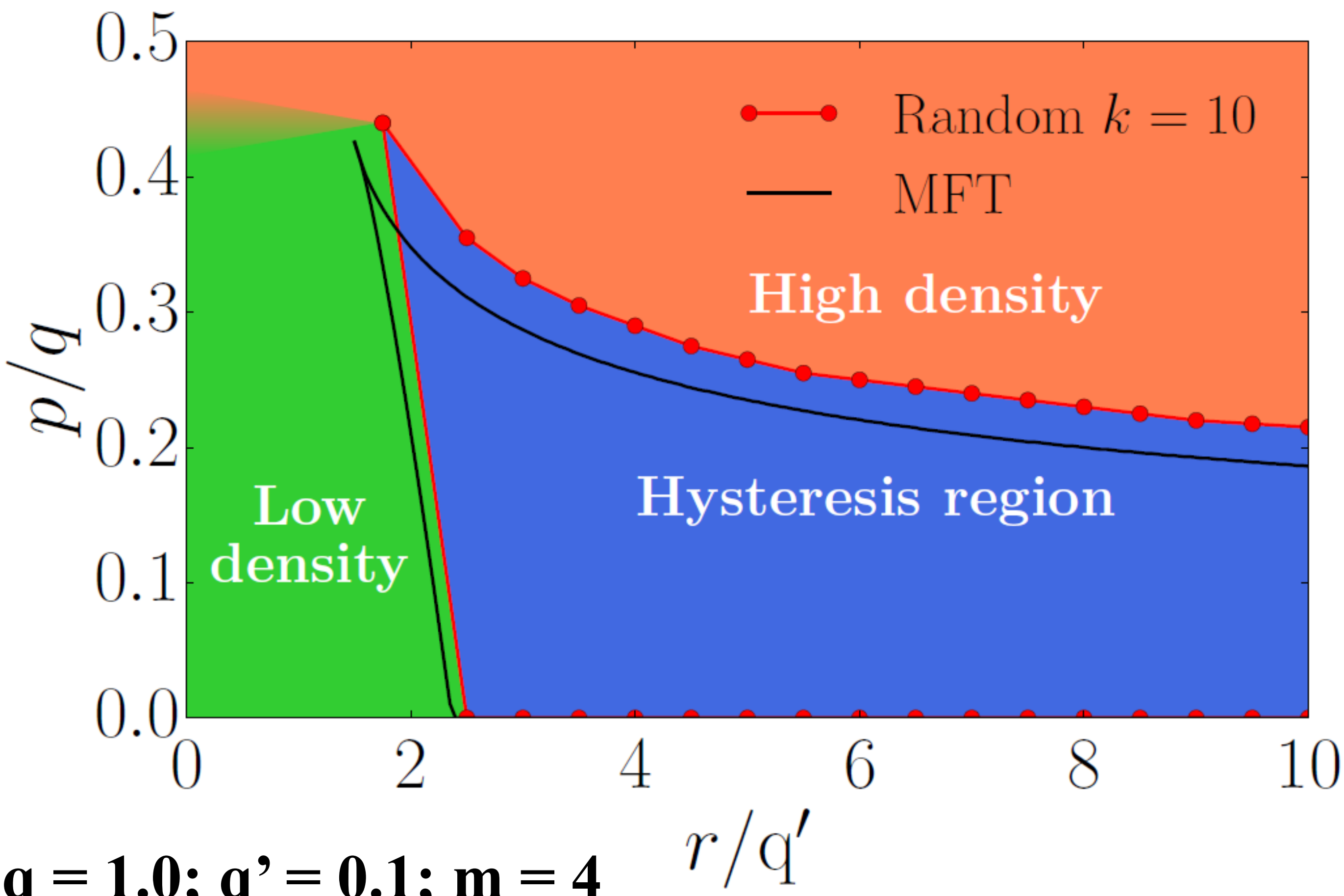


square lattice: 1024 × 1024

p = 0; q’ = 1.0

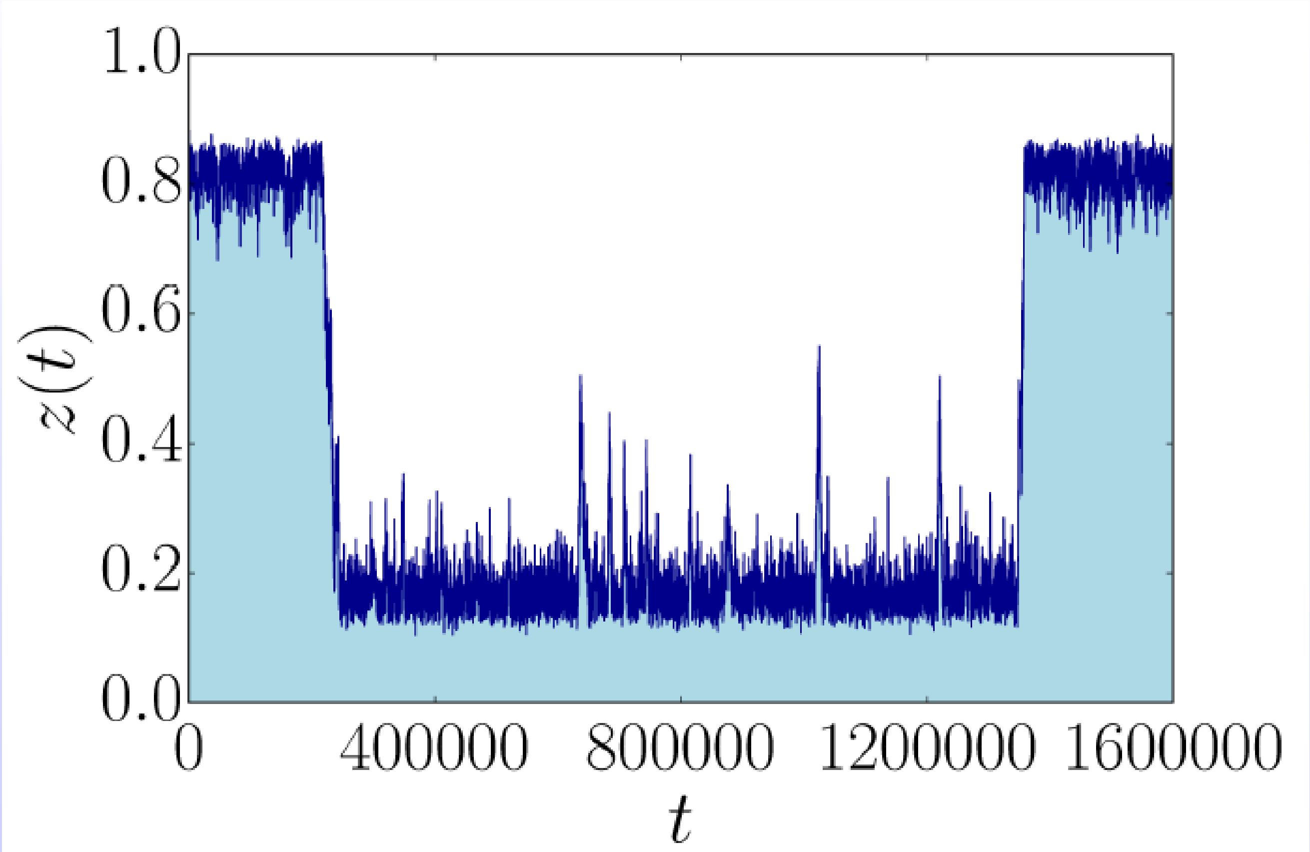
Phase Diagram

regular random graph with k = 10



q = 1.0; q' = 0.1; m = 4

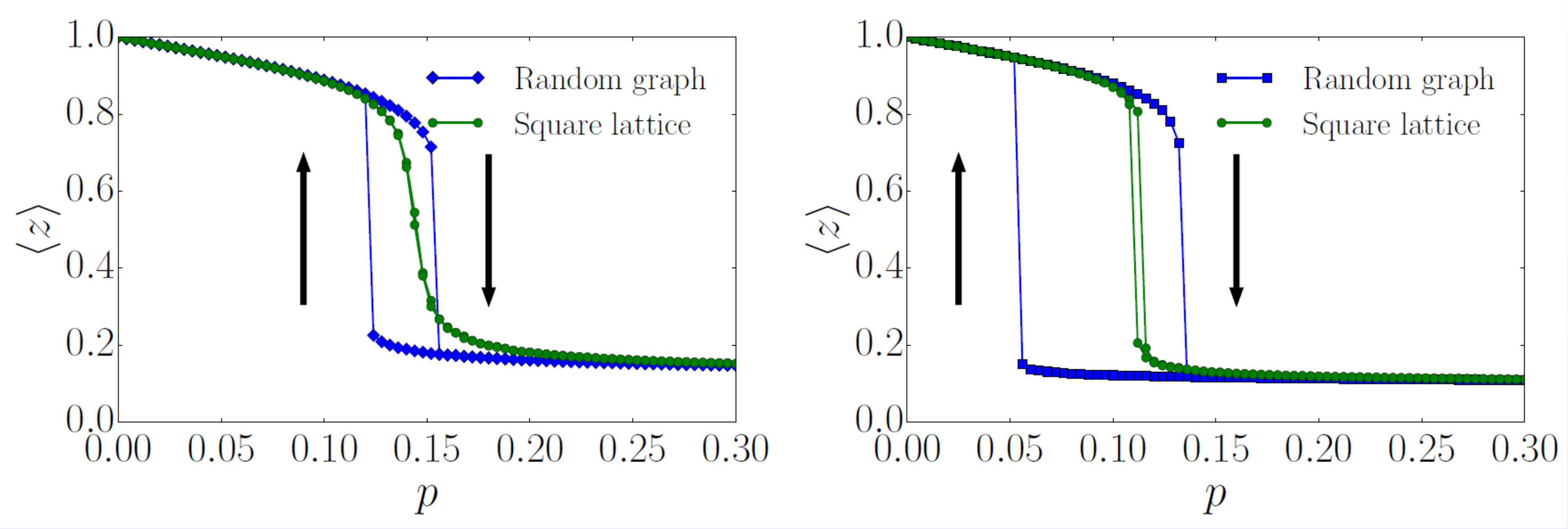
Phase switching



$p = 0.1065$, $r = 0.95$, $q = 1.0$, $q' = 0.1$ and $m = 1$

Hysteresis

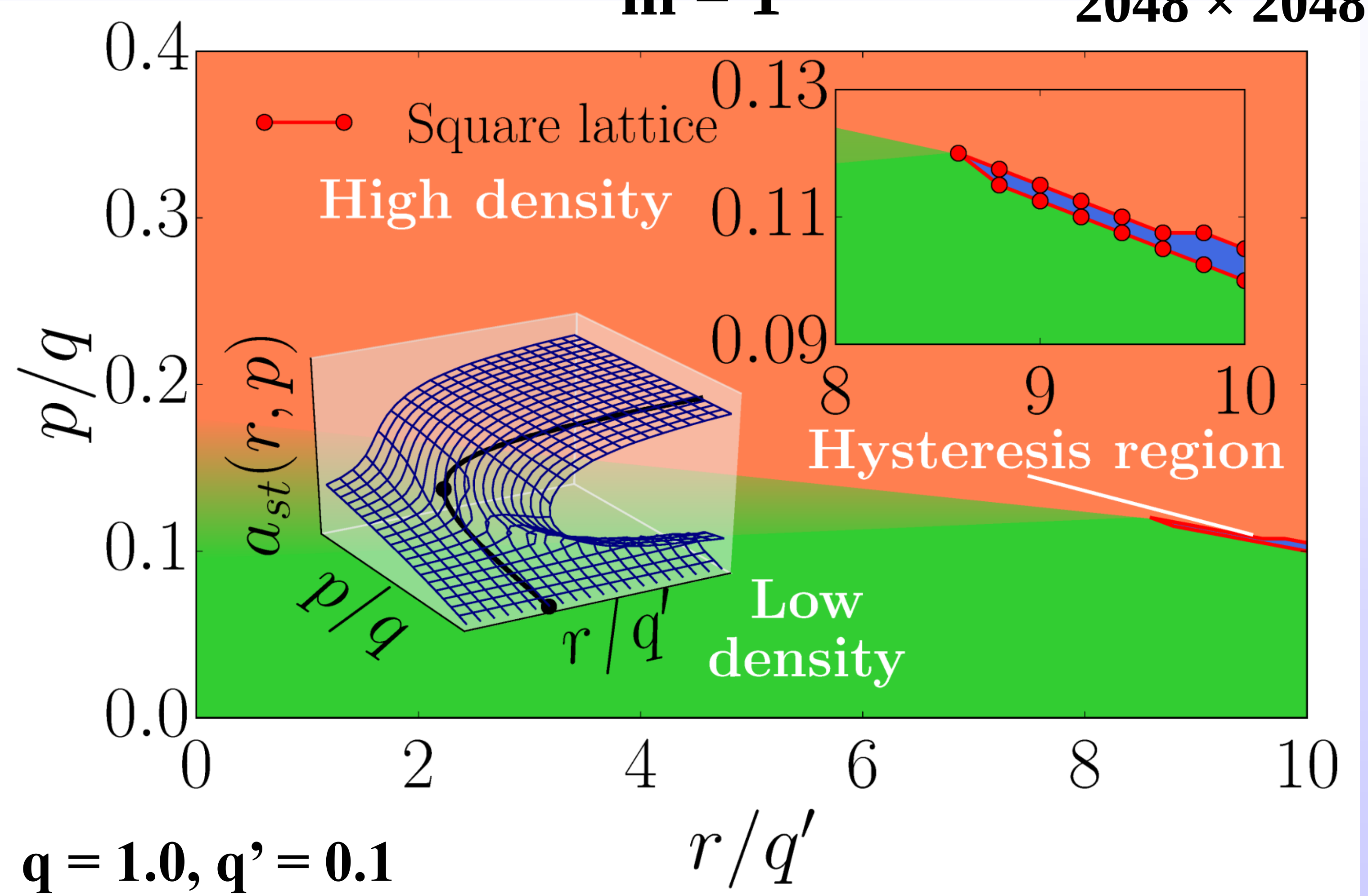
512 × 512 square lattice ; q = 1.0; q’ = 0.1; m = 1



r = 0.7 **r = 1.0**

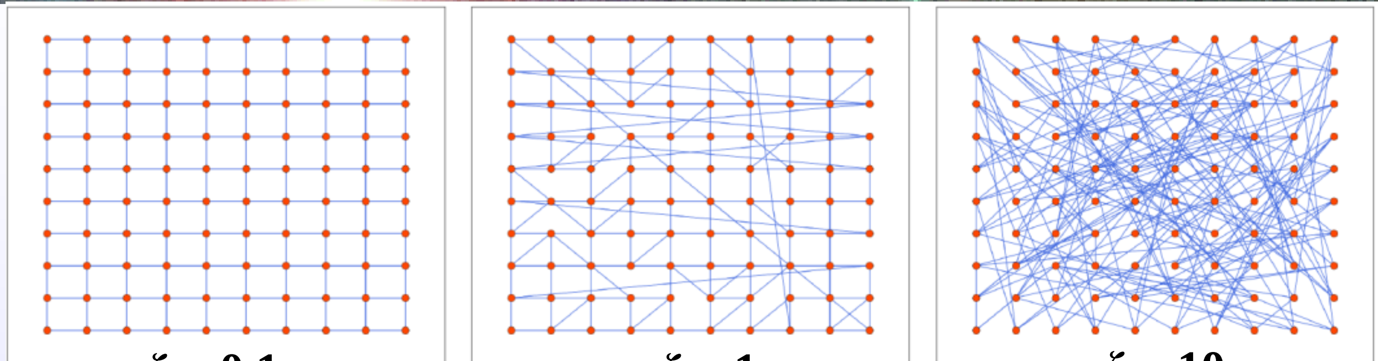
Phase Diagram

m = 1 2048 × 2048



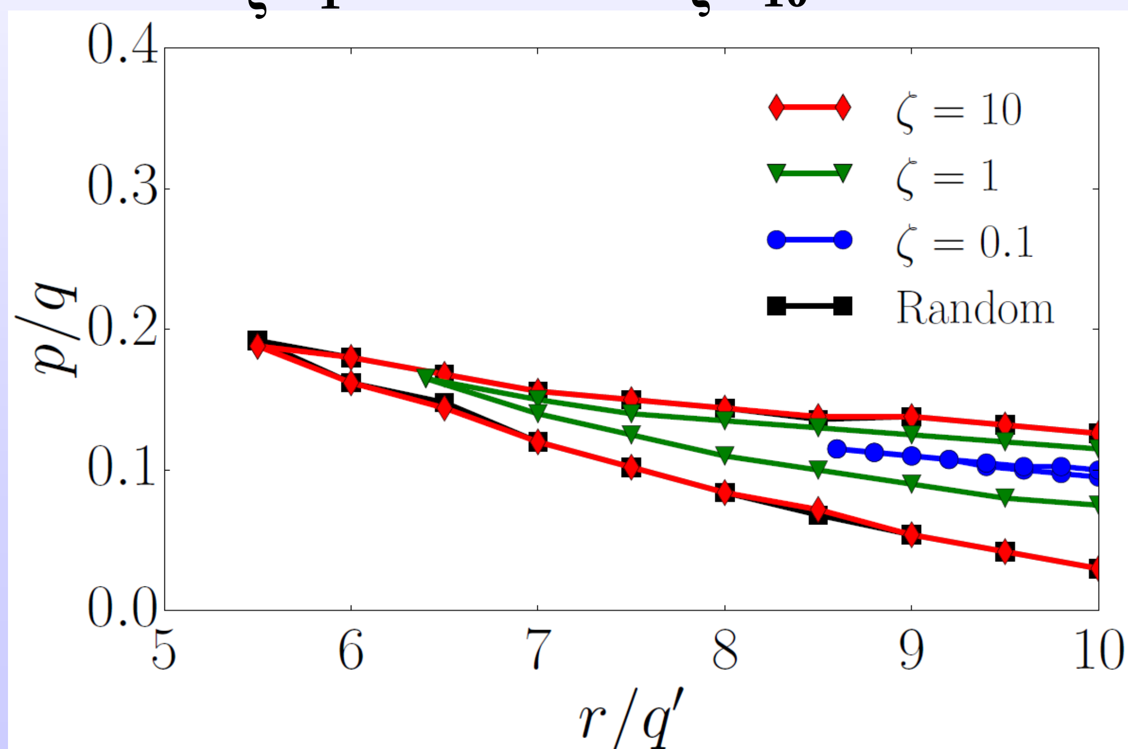
q = 1.0, q’ = 0.1

Going from a square lattice to a random one

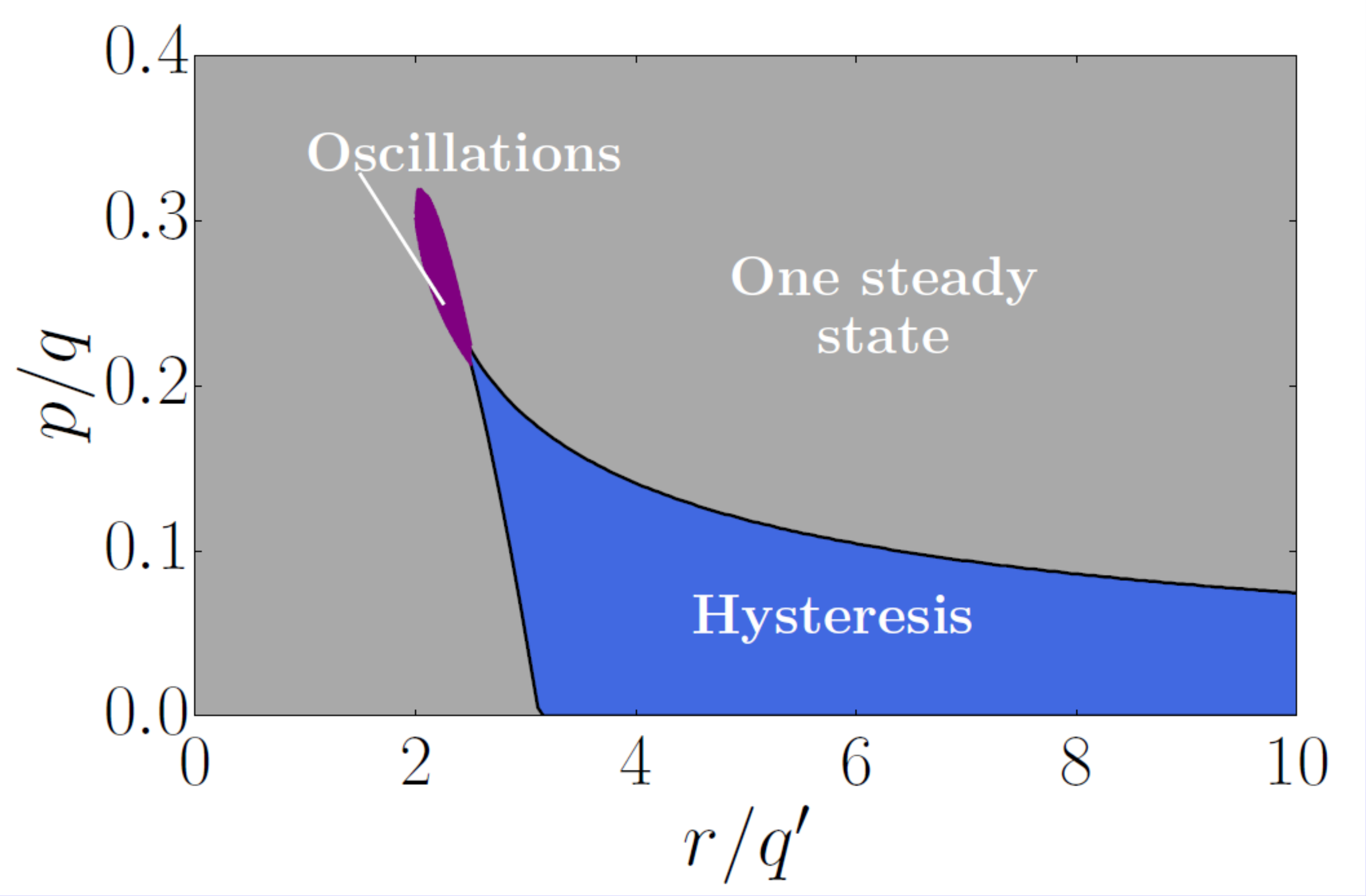


ζ = 0.1 ζ = 1 ζ = 10

q = 1.0
q’ = 0.1
m = 1



Oscillatory behavior

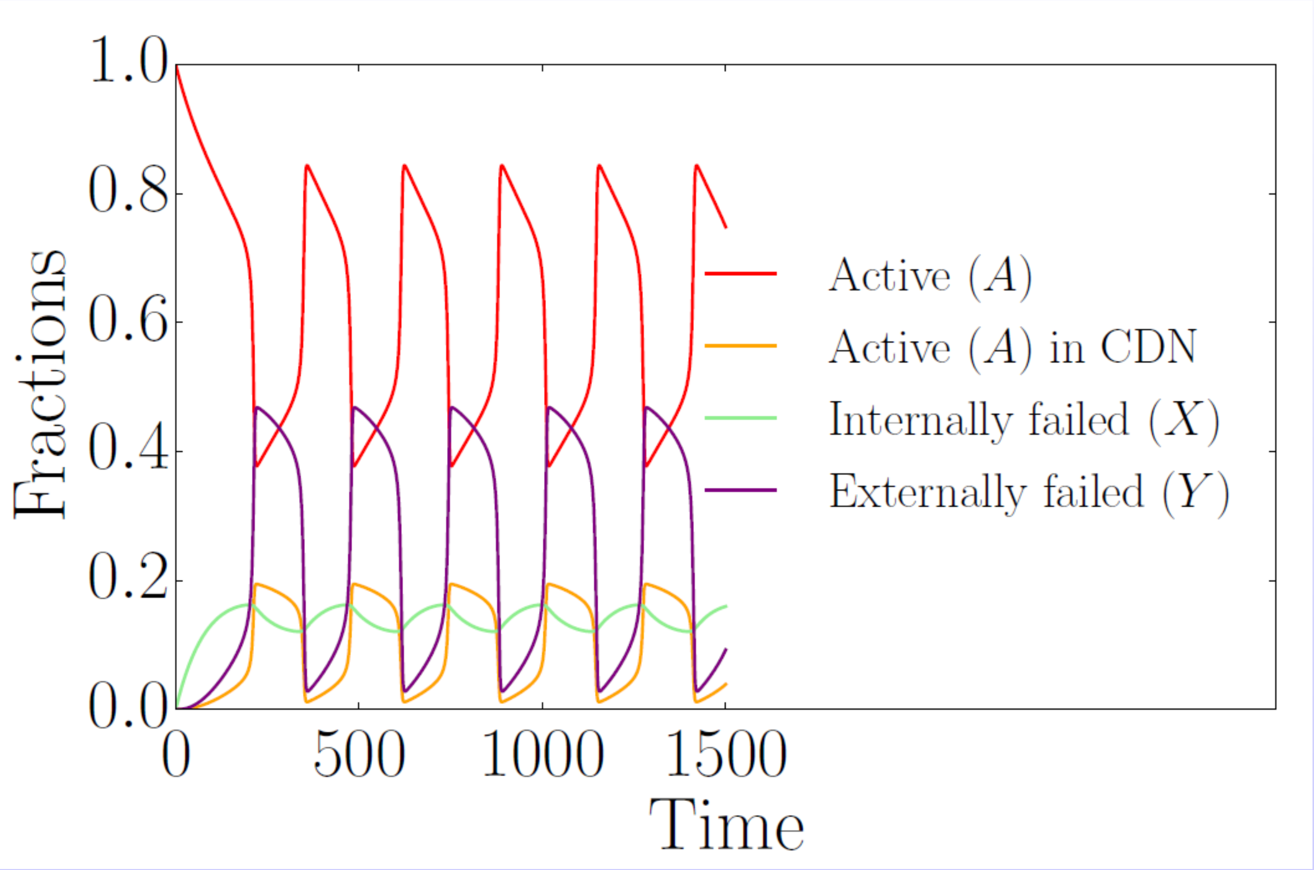


q’ > q

k = 4, q = 0.01, q‘ = 1.0 and m = 1

Oscillatory behavior

mean field calculation for
p/q = 19/81; q = 0.01; r/q’ = 3125/1296; q’ = 1.0; m=1



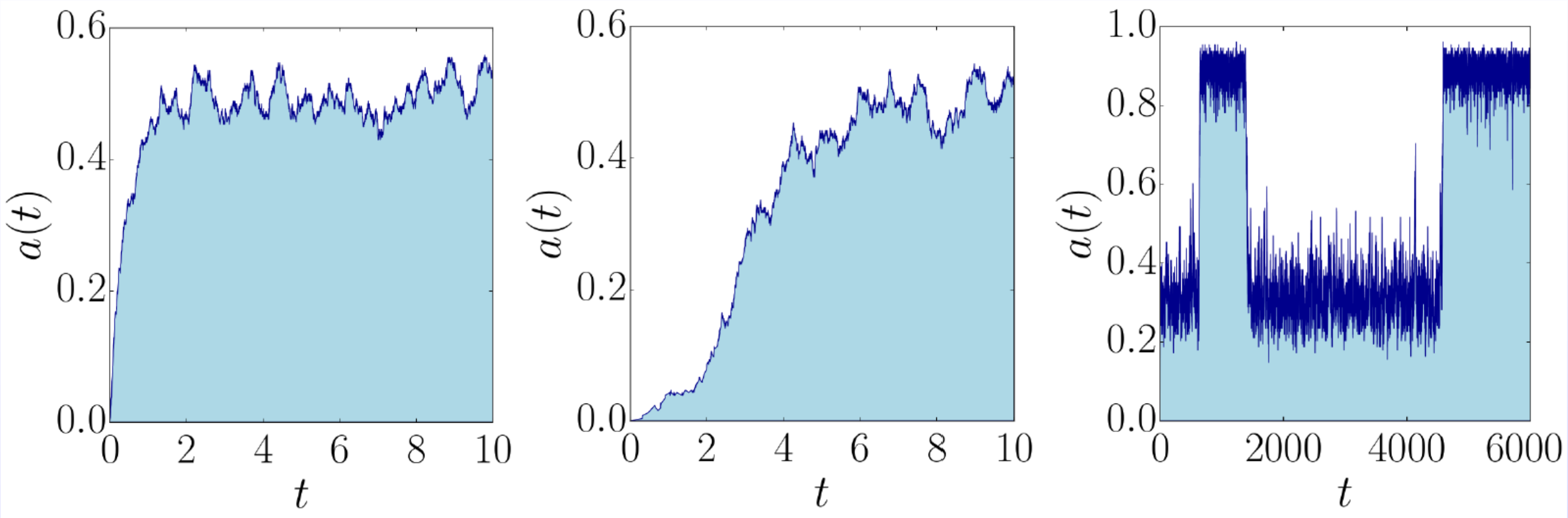
Oscillatory State

regular random
graph with k = 10
N = 1000

p = 0.007
r = 1.8
q = 0.01
q’ = 1.0
m = 4

Only three posible scenarios

m = k m = k - 1 m < k - 1



Schlögl I
Contact process
SIS model
Reggeon field theory
Directed percolation

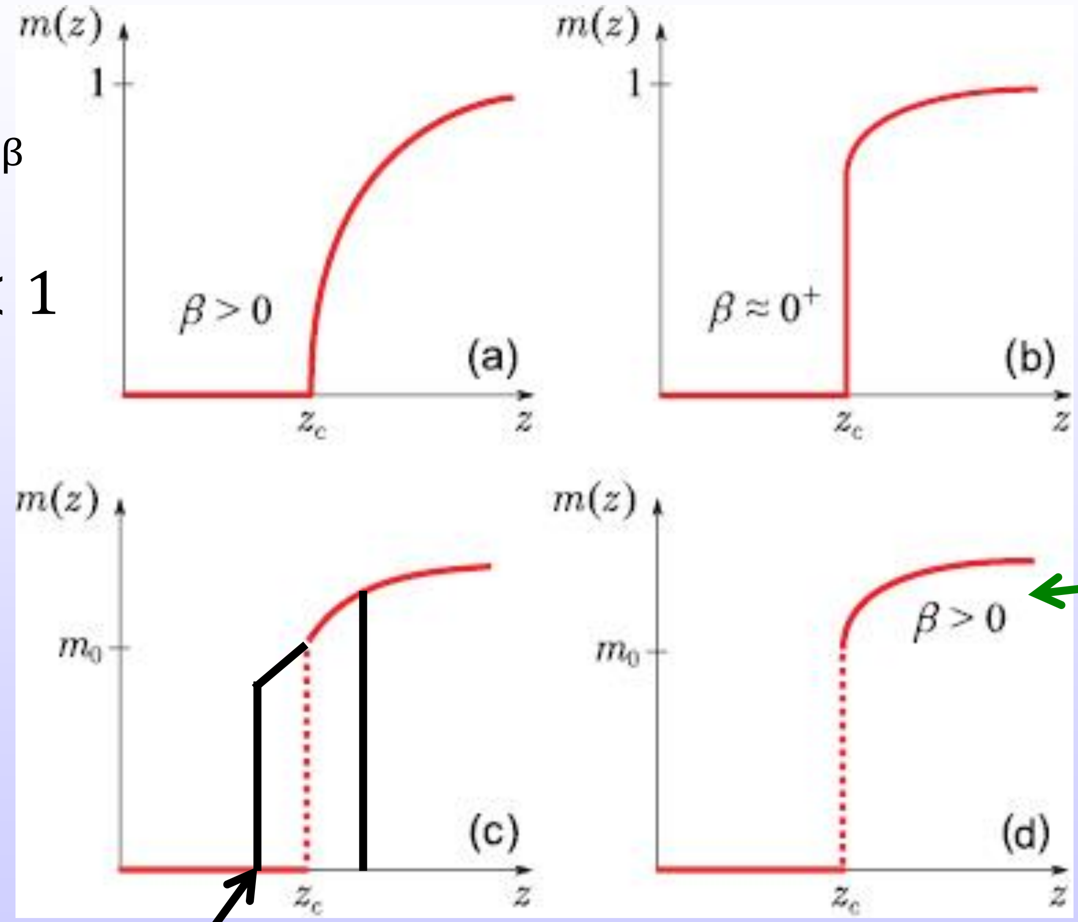
Schlögl II
Quadratic contact process
General contact process
Threshold models of complex contagions

L. Böttcher, J. Nagler, H.J.H., Phys. Rev. Lett. 118, 088301 (2017)

Different types of transitions

$m \sim (z - z_c)^{\beta}$

$\frac{5}{36} < \beta < 1$



second order

= continuous

hybrid

first order

= discontinuous

hysteresis

Summary

- Global budgets produce total infection (pandemics).
- Requiring contact to a supply center produces sudden infection jumps (first order transition) and subsequent pandemics.
- If also spontaneous infection occurs metastability is observed ($m < k-1$).
- If recovery from spontaneous infection is slower oscillations can be found ($q' > q$).
- Targetted recovery confines outbreak on lattice with local connections.

Thank you