An analytical expression for the exit probability of the q-voter model in one dimension

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We present an approximation that is able to give an analytical expression for the exit probability of the q-voter model in one dimension. This expression gives a better fit for the more recent data about simulations in large networks [1], and as such, departs from the expression $\frac{\rho^q}{\rho^q+(1-\rho)^q}$ found in papers that investigated small networks only [2, 3, 4]. The approximation consists in assuming a large separation on the time scales at which active groups of agents convince inactive ones and the time taken in the competition between active groups. Some interesting findings are that the limit for large q is $\frac{2\rho^q(1-\rho)^{q-1}+\rho^{2q-2}}{2\rho^{q-1}(1-\rho)^{q-1}+\rho^{2q-2}+(1-\rho)^{2q-2}}$; for $q = 2$ we still have $\frac{\rho^2}{\rho^2+(1-\rho)^2}$ as the exit probability and for $q > 2$ we can obtain a lower order approximation of the form $\frac{\rho^s}{\rho^s+(1-\rho)^s}$ with s varying from q for low values of q to $q - \frac{1}{2}$ for large values of q. As such, this work can also be seen as a deduction for why the exit probability $\frac{\rho^q}{\rho^q+(1-\rho)^q}$ gives a good fit, without relying on mean field arguments or on the assumption that only the first step is non deterministic, as q and $q - \frac{1}{2}$ will give very similar results when $q \to \infty$.

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