The Price of Anarchy on a Diamond Lattice Network

A.Rose

Department of Mathematical Sciences, University of Nottingham, UK
(email: pmxar3@nottingham.ac.uk)

If a society is comprised of uncoordinated individuals, one might expect each individual to attempt to optimise their personal gain whilst disregarding the welfare of the society as a whole. This often results in the group welfare, the aggregate gain of all the individuals, being suboptimal. This is known as the *price of anarchy* (POA). As a simple theoretical paradigm for such a system, we consider routing traffic through a congestible network. The network is comprised of two types of edges whereby the cost of traversing an edge is either constant or proportional to the congestion of traffic on that edge. Edges of the latter type are the more desirable to network users, but over-usage will result in a sub-optimal flow of traffic. This phenomenon can be quantified by the POA, whereby a value of unity corresponds to the users using the network completely efficiently even when uncoordinated, i.e. there being no price of anarchy, and a value greater than unity corresponds to inefficiency of uncoordinated network users. We consider traffic flowing through a network on a plane with a diamond lattice structure, whereby a given proportion of randomly-placed edges have the variable cost. We analyse the effect on the POA of varying the proportion of variable edges, which we conduct over lattices of differing height to width aspect ratio. For lattices with aspect ratio 1:1, it is known that as this proportion increases, the value of the POA rises steadily from unity before abruptly falling back to unity once the percolation threshold is exceeded. Altering the aspect ratio so that the paths from one side of the lattice to the other are relatively short, produces a rippling effect on the POA for when the proportion of variable edges is less than the percolation threshold. Moreover, the number of local maxima is equal to the path length. We show that local maxima occur when the number of variable edges within a path matches the number of paths in the entire network having this property. The emergence of the structure in the POA occurs therefore at a particular confluence of the microscopic structure of the paths and the macroscopic structure of the network. The robustness of this result will be considered for different tilings of the network's microscopic structure and its macroscopic aspect ratio.