Scaling in the Diffusion Limited Aggregation Model

A.Yu. Menshutin, L.N. Shchur

Landau Institute for Theoretical Physics and Scientific Center in Chernogolovka, 142432 Chernogolovka, Russia

Diffusion limited aggregation model is a bright example of a system that exhibits critical properties as a result of non-equilibrium growth process. Although it was introduced almost thirty years ago by Witten and Sander, it is still of great interest because of its computational and theoretical complexity. Structures grown by DLA look very similar to those found in nature and society, for example, ice crystals on window, mineral dendrites on the surfaces of limestones, colony of bacteria etc. Different well-established theories explain why various systems behave very similarly near a critical point. But understanding of nonequilibrium critical phenomena is still an open problem. In this research we study the question of scaling behavior of DLA clusters by extensive numerical simulation. We build clusters with different number n of axes of the anisotropy field,varying from $n = 3$ to $n = 8$, as well as off-lattice clusters. We also vary so-called noise reduction level m, which physically could be considered as a quantity, inversely proportional to adhesion. As the result we obtain a phase-like diagram that contains two regions in the phase plane (n,m) which correspond to the fractal crystals (D=3/2) and to the random fractals (D=1.710). Thus, lattice and off-lattice models belong to two distinct classes.

We also study the convergence of the DLA model. We suggest a simple but complete picture of DLA growth based on analyzing the probability density function $P(r, N)$ for the next particle to be attached at a distance r from the origin. We propose a scale-invariant form for the function $P(r, N)$

$$P(r, N) = \frac{1}{R_{\rm dep}} f\left(\frac{r}{R_{\rm dep}(N)}\right).$$

and check our assumptions numerically. It immediately follows from our theory that there is only one scaling exponent D and there is no multiscaling in an asymptotically large DLA cluster.

[1] A.Yu. Menshutin and L.N. Shchur, Comp. Phys. Comm. **182** 1819 (2011)

[2] A.Yu. Menshutin, Phys. Rev. Lett. **108**, 015501 (2012)