Logistic Growth and Ergodic Properties of Urban Forms

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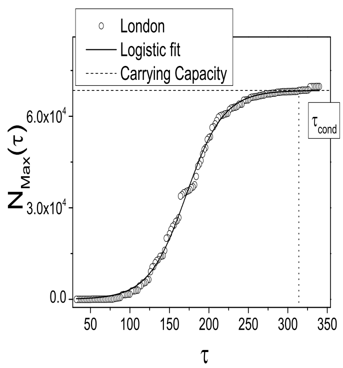
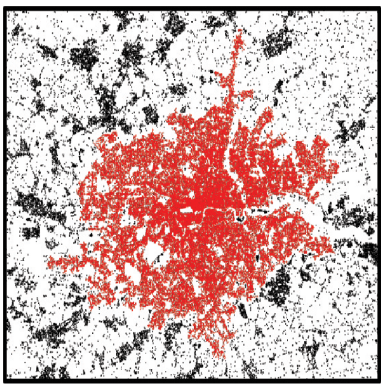
Urban morphology has presented significant intellectual challenges to mathematicians and physicists ever since the eighteenth century, when Euler first explored the famous Königsberg bridges problem. Many important regularities and allometries have been observed in urban studies, including Zipf's law and Gibrat's law, rendering cities attractive systems for analysis within statistical physics. Nevertheless, a broad consensus on how cities and their boundaries are defined is still lacking. Applying percolation theory to the street intersection space for over one hundred between the largest cities in the UK and in California, we show that growth curves for the maximum cluster size collapse to a single curve, namely the logistic (see Fig.). Subsequently, by introducing the concept of the condensation threshold, we show that the structural fringe of the city can be defined mathematically in a universal way. This is achieved by obtaining the parameters at the point at which a condensation phenomenon is observed. Such universality in the spatial properties of cities allows us to study and discuss systematically some of the allometries that are present in cities, and prompts us to look at the temporal behaviour of important properties of urban street networks, thus casting light on the concept of ergodicity as related to urban street networks [1].





[1] AP Masucci et al., submitted, http://arxiv.org/abs/1504.07380 (2015).