SECOND LAW OF THERMODYNAMICS FOR RANDOM MODELS OF WEALTH EXCHANGE

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We discuss the second law of thermodynamics for systems of economical agents who exchange their money in binary 'collisions' like e.g. in Drăgulescu-Yakovenko model (see e.g. [1]), where exchange between two agents with initial amounts of wealth x and y results in $x, y \to x', y'$ with

$$x' = \epsilon(x + y), \qquad y' = (1 - \epsilon)(x + y),$$

where $0 < \epsilon < 1$ is a random number with distribution $D(\epsilon)$. The evolution of wealth distribution $p(x)$ in such models is governed by a nonlinear Boltzmann-type equation. But, as noticed already in [1], the Boltzmann-Shannon entropy $S_B(p) = -\int_0^\infty dxp(x)\ln p(x)$ does not always increase with time because the elementary act of exchange is in fact not a mechanical process, but rather an irreversible one. Indeed, final amounts of wealth can be obtained by maximizing Cobb-Douglas utility function $U = x^\epsilon y^{1-\epsilon}$ at $x + y = \text{const}$, which indicates possible presence of some hidden 'intelligent' agents, similar to Maxwell's demon, who actually control exchanges.

We approach this problem using the two-particle distribution function formalism [2] and obtain the second law in the form of a Clausius-type inequality [3], which contains the change in entropy $S_B(p)$ along with some 'heat' flow to an external reservoir. This heat transfer appears because the wealth exchange may be viewed as an information erasure by thermal randomization, since as a result of exchange different distributions of the fraction $\xi = x/(x + y)$ for pairs of agents are replaced by the fixed one, $D(\xi)$. For $D(\epsilon) \sim \epsilon^a(1 - \epsilon)^a$ the Clausius inequality results in the H-theorem with the modified 'entropy' functional $S(p) = S_B(p) + a\int_0^\infty dx\, p(x)\ln x$. Additional term in $S(p)$ may be regarded as a contribution due to auxiliary rational agents, not specified explicitly, who erase information during exchanges by maximizing Cobb-Douglas utility functions.

[1] V.M. Yakovenko, J. B. Rosser Jr., Rev. Mod. Phys. **81**, 1703 (2009).

[2] S.M. Apenko, Phys. Rev. E **87** 024101 (2013).

[3] S.M. Apenko, Physica A **414**, 108 (2014).