McKean-Vlasov limit for interacting systems with simultaneous jumps

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In this work we consider systems of N weakly interacting diffusions with jumps, having the peculiar feature that the jump of one component may induce simultaneous jumps of all others. Models belonging to this class have been proposed for the dynamics of neuronal systems, see [2, 4, 5], and their limiting ($N \to +\infty$) behavior has been studied only for special cases in the positive half-line. We prove propagation of chaos and derive the corresponding McKean-Vlasov equation in a more general d-dimensional framework. Unlike existing approaches in neuronal models, we show that classical tools can be profitably adapted to treat this class of systems with a good degree of generality. We apply our method to three classes of systems: systems of diffusions with simultaneous jumps where the coefficients satisfy global Lipschitz-type conditions, extending the model presented in [3]; systems of diffusions with jumps where the drift coefficient presents a non-standard behavior, similar to the one presented in [1]; particular systems of equations with jumps where the jump rate has a local Lipschitz behavior, generalizing the model in [5].

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