MULTIOBJECTIVE OPTIMIZATION AND SYSTEMS POISED TO CRITICALITY

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Multiobjective (or Pareto) optimization deals with the optimization of several target functions at the same time. The solution to such problems are the most optimal tradeoffs between the different targets and usually comprise a collection of Pareto optimal designs instead of one global optimizer. Designs implementing this tradeoff constitute the Pareto front, a geometric object deeply related to the Gibbs surface of thermodynamic systems: both encode phase transitions and critical points according to the same mathematical criteria that involve their shape and differential geometry [1]. (Consistently, when treating thermodynamics as a Pareto optimization problem – through simultaneous energy minimization and entropy maximization – all the relevant phenomenology of phase transitions and criticality is parsimoniously recovered.) We term Pareto selective forces to any algorithmic procedure (artificial or natural) that copes with this simultaneous optimization by evolving arbitrary populations towards some Pareto front. When this front meets a trivial geometric condition, Pareto selective forces always drive populations towards a critical state that separates the two phases of a first order phase transition [2]. It is not required to tune any external parameter for this spontaneous critical tuning to take place. This criticality is, indeed, an emergent property of the Pareto optimal designs as an ensemble. We illustrate this phenomenon with actual computational examples of Pareto optimal complex networks [3] and compare our findings with well established critical systems – e.g. thermodynamic critical points.

[1] Seoane LF, Solé RV, A multiobjective optimization approach to statistical mechanics. arXiv preprint: http://arxiv.org/abs/1310.6372

[2] Seoane LF, Solé RV, Systems poised to criticality through Pareto selective forces. In preparation.

[3] Seoane LF, Solé RV, Phase transitions in Pareto optimal complex networks. In preparation.