Emergence of Periodic Behaviours from Randomness

J N Pickton[1], K I Hopcraft, E Jakeman

School of Mathematical Sciences, University of Nottingham, Nottingham, NG7 2RD, UK
E-mail: pmxjp5@nottingham.ac.uk[1]

Periodic behaviours can be described with great power and economy using the simple mathematical machinery associated with wave phenomena. However periodic effects can also be observed in collections of discrete objects, be they individuals sending emails, fire-flies signalling to attract mates, synapses firing in the brain or photons emerging from a cavity. The identification and origin of what constitutes the wave-like property becomes more difficult to interpret and identify in these instances but can be most simply exemplified by consideration of non-interacting particles moving randomly on a network forming N nodes in a closed loop. Specifically the population dynamics describing the number of particles at a node is a familiar stochastic birth-death process, augmented by particles jumping randomly at rate r to adjacent nodes in either direction. This can result in the emergence of periodic behaviours which occur because of the interaction between the dynamics of the particles and the spatial structure through which they move. For this to happen we show that the network must consist of three or more nodes and the particles must have a preferred direction of jumping. Moreover there are three very different classes of collective behaviour of the populations at the nodes which emerge depending on the value of the birth-rate μ. The first occurs when $0 < \mu < r(1 - r\cos(2\pi/N))$ in which case the particles become uniformly distributed across all nodes in the network. The second occurs when $r(1-\cos(2\pi/N)) < \mu < 2r$ whereupon the populations become localised and propagate coherently around the network, forming a travelling wave-packet. The last is last regime is $\mu > 2r$ when the all particles collapse into a single node and no longer propagate around the network. Thus distinct coherent structures can emerge purely through random interactions.