DECADENCE and FALL of the ACADEMIES

Claudio Parmeggiani (c.parmeggiani@campus.unimib.it)

University of Milano-Bicocca (UNIMIB) - Italy

Here an "*academy*" is a generic group of people (the members) where new members are chosen (co-opted) by the old ones. Then an academy is eventually a learned society, but also a political party or the group of partners of a counseling firm (and so on).
If $N(t)$ is the number of members at time t, we can assume that only N_A are "*bright*" while the others N_B ($= N - N_A$) are "*average*" or poor. $\mu N(t)$ members ($0 < \mu < 1$) leave spontaneously (or by natural causes) the group at time t; consequently $\alpha N(t)$ new members has to be co-opted by the remaining ones; obviously α is $\geq \mu$, if not the academy will evaporate.
Then we shall assume that the B-members (almost) always co-opt new average members ($\alpha N_B(t)$, in number) while the A-members generally chose new bright ones, but sometimes they do a mistake and select average ones; in this case there are only $(\alpha - \delta)\, N_A(t)$ ($0 < \delta < \alpha$) new bright and $\delta N_A(t)$ new average members, at time t.
The academy's dynamics is described by a couple of differential equations:

$$N'_A(t) = (\alpha - \delta)\, N_A(t) - \mu N_A(t)$$
$$N'_B(t) = \delta N_A(t) + \alpha N_B(t) - \mu N_B(t)$$

and $N'(t) = \alpha N(t) - \mu N(t)$. If, for example, $N(t) = const$ and $\alpha = \mu$, the bright members exponentially disappear: $N_A(t) = N_A(0)e^{-\delta t}$. It is possible to devise some modification of the co-optation procedures (co-optation committees, objectives evaluations) but we can only alter the value of δ and the B-members will again rapidly predominate.
A different, quite effective (but not so popular) approach is possible: we can prescribe that a *fixed* number of members, $\nu N(t)$ ($0 < \nu < 1$), *must* be expelled from the academy, at time t; now it is reasonably to assume that A-members (almost) always expel the average ones while B-members generally dismiss the bright ones (if someone is left); we arrive at:

$$N'_A(t) = (\alpha - \delta - \mu)\, N_A(t) - \nu N_B(t)$$
$$N'_B(t) = (\delta - \nu)\, N_A(t) + (\alpha - \mu)\, N_B(t)$$

and $N'(t) = (\alpha - \mu - \nu)\, N(t)$. Here $t < T_A(T_B)$ where $T_A(T_B)$ is the first zero of $N_A(N_B)$. If $N(t) = const$, $\alpha = \mu + \nu$ (and $\nu > \delta$), $\frac{N_A(t)}{N} = \left(\frac{N_A(0)}{N} - \frac{\nu}{2\nu-\delta}\right) e^{(2\nu-\delta)t} + \frac{\nu}{2\nu-\delta}$ and $N\left(\frac{\nu}{2\nu-\delta}, \frac{\nu-\delta}{2\nu-\delta}\right)$ is a *critical point*. If $\frac{N_A(0)}{N} > \frac{\nu}{2\nu-\delta}$ ($< \frac{\nu}{2\nu-\delta}$), N_A increase (decrease); if $\frac{N_A(0)}{N} = \frac{\nu}{2\nu-\delta}$, N_A and N_B are constants (they don't leave the critical point).
Therefore if, at some time, $\frac{N_A}{N}\frac{2\nu-\delta}{\nu} < 1$ the academy "*decade and fall*", inexorably; otherwise the academy "*rise*".