

## SYNCHRONISATION IN A SYSTEM OF POLYA URNS INTERACTING THROUGH A MEAN-FIELD REINFORCEMENT

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In this presentation, we consider a finite system of interacting stochastic dynamics. Each individual's dynamics consists in a two colours urn with a kind of Polya reinforcement scheme [8]: each urn's reinforcement depends both on the urn's content and on the average content of all urns (mean-field). The underlying network is thus the complete graph. Urn's states are updated simultaneously. In [1] we prove that the urns synchronise [6,7] almost surely towards the same random limit, for all urns, as the time goes to infinity. Contrary to what is proven for time-cyclic systems, like the Kuramoto model [4,5], the emergence of this collective behaviour is not a thermodynamical effect. We will explain, it comes indeed from the linear reinforcement scheme through the mean-field. When a large number of urns interact, a normal approximation for the system is stated. Generalised models will be presented as for instance emphasised in [2]. This is an ongoing joint work with P. Dai Pra, I. Minelli, I. Crimaldi and N. Sahasrabudhe. In particular, fluctuations are considered in [3].

[1] P. Dai Pra, P.-Y. Louis and I. G. Minelli, *Synchronization via interacting reinforcement*. J. Appl. Probab., 51(2) :556–568 (2014).

[2] P.-Y. Louis, I. Minelli, *Synchronization in Interacting Reinforced Stochastic Processes*. Book's Chapter, in *Probabilistic Cellular Automata, Theory, Applications and Future perspectives* (to be published in 2016).

[3] I. Crimaldi, P. Dai Pra and I. G. Minelli, *Fluctuation Theorems for Synchronization of Interacting Polya's urns*, arXiv:1407.5043 (2014).

[4] J. A. Acebrón, L. L. Bonilla, C. J. Pérez Vicente, F. Ritort, and R. Spigler, *The Kuramoto model: A simple paradigm for synchronization phenomena*, Rev. Mod. Phys. 77, 137 (2005).

- [5] L. Bertini, G. Giacomin, K. Pakdaman, *Dynamical aspects of mean field plane rotators and the Kuramoto model*, J. Stat. Phys., 138, 1-3, 270–290 (2010).
- [6] S. Strogatz, *Sync: How Order Emerges From Chaos In the Universe, Nature, and Daily Life* (2003).
- [7] Pikovsky, Rosenblum and Kurths, *Synchronization: a universal concept in nonlinear sciences* (2001).
- [8] H. M. Mahmoud, *Polya Urn Models*, (2008).