Exact quasiclassical asymptotics beyond Maslov canonical operator and quantum jumps nature

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We suggest a new asymptotic representation for the quantum averages: (1) $\langle i,t,\mathbf{x}_0;\hbar\rangle = \int_{\mathbb{R}^n} x_i \left|\Psi(\mathbf{x},t,\mathbf{x}_0;\hbar)\right|^2 d^{\,n}x$ with position variable with well localized initial data,i.e.$\langle i,0,\mathbf{x}_0;\hbar\rangle \simeq x_{0i}, i=1,...,n$.such quasiclassical asimptotic very important from point of view of the "quantum jumps" problem, well known in modern quantum mechanics.The existence of such jumps was required by Bohr in his theory of the atom. He assumed that an atom remained in an atomic eigenstate until it made an instantaneous jump to another state with the emission or absorption of a photon. Since these jumps do not appear to occur in solutions of the Schrodinger equation, something similar to Bohr's idea has been added as an extra postulate in modern quantum mechanics. The question arises whether an explanation of these jumps can be found to result from a solution $\Psi(\mathbf{x},t,\mathbf{x}_0;\hbar)$ of the Schrödinger equation alone without additional postulates.The canonical physical interpretation of these asymptotics shows that the answer is "yes",see also [1].

Theorem. Let $\Psi(t,\mathbf{x})$ be the solution of the Schrödinger equation in $\mathbb{R}^n$ with initial data: (2) $\Psi(0,\mathbf{x}) \simeq \delta(\mathbf{x}-\mathbf{x}_0)$ and potential $V(\mathbf{x},t)$.We assume that $V(\cdot,t):\mathbb{R}^n \to \mathbb{R}$ is an polynomial function in variable $\mathbf{x}=(x_1,x_2,...,x_n)$,i.e.$V(\mathbf{x},t) =$

$\sum_{\|\alpha\|\leqslant m} g_\alpha(t)\, x^\alpha, \alpha=(i_1,...,i_n), \|\alpha\| = \sum_r i_r, i=1,...,n, x^\alpha = x_1^{i_1}x_2^{i_2}...x_n^{i_n}$.Let $\mathbf{u}(\tau,t,\boldsymbol{\lambda},\mathbf{x}) = (u_1(\tau,t,\boldsymbol{\lambda},\mathbf{x}),...,u_n(\tau,t,\boldsymbol{\lambda},\mathbf{x}))$ be the solution of the linear differential boundary problem: (3) $d^{\,2}\mathbf{u}(\tau,t,\boldsymbol{\lambda},\mathbf{y},\mathbf{x})/d\tau^2 = \mathbf{Hess}[V(\boldsymbol{\lambda},\tau)]\,\mathbf{u}^{\mathbf{T}}(\tau,t,\boldsymbol{\lambda},\mathbf{y},\mathbf{x}) + \mathbf{V}'(\boldsymbol{\lambda},\tau)^{\mathbf{T}}, \mathbf{u}(0,t,\lambda,\mathbf{y},\mathbf{x}) = \mathbf{y}, \mathbf{u}(t,t,\lambda,\mathbf{y},\mathbf{x}) = \mathbf{x}$.Here $\mathbf{y},\mathbf{x}\in\mathbb{R}^n, \boldsymbol{\lambda} = (\lambda_1,...,\lambda_n)$,
$\mathbf{u}^{\mathbf{T}}(\tau,t,\boldsymbol{\lambda},\mathbf{y},\mathbf{x}) = (u_1(\tau,t,\boldsymbol{\lambda},\mathbf{y},\mathbf{x}),...,u_n(\tau,t,\boldsymbol{\lambda},\mathbf{y},\mathbf{x}))^{\mathbf{T}}, \mathbf{V}'(\boldsymbol{\lambda},\tau) = (\partial V(\mathbf{x},\tau)/\partial x_1,...,\partial V(\mathbf{x},\tau)/\partial x_n)_{\mathbf{x}=\boldsymbol{\lambda}}$

and $\mathbf{Hess}[V(\boldsymbol{\lambda},\tau)]$ the Hessian of the function $V(\cdot,\tau):\mathbb{R}^n\to\mathbb{R}$ at point $\mathbf{x}=\boldsymbol{\lambda}$, i.e. $\mathbf{Hess}_{ij}[V(\boldsymbol{\lambda},\tau)] = \left[\partial^2 V(\mathbf{x},\tau)/\partial x_i\partial x_j\right]_{\mathbf{x}=\boldsymbol{\lambda}}$.Let $\mathbf{S}(t,\boldsymbol{\lambda},\mathbf{y},\mathbf{x}) =$
$\int_0^t \widetilde{\triangle}(\dot{\mathbf{u}}(\tau,t,\boldsymbol{\lambda},\mathbf{y},\mathbf{x}),\mathbf{u}(\tau,t,\boldsymbol{\lambda},\mathbf{y},\mathbf{x}),\tau)\,d\tau$ where $\widetilde{\triangle}(\dot{\mathbf{u}}(\tau),\mathbf{u}(\tau),\tau) = m\dot{\mathbf{u}}^2(\tau)/2 -$
$\widehat{V}(\mathbf{u}(\tau),\tau), \widehat{V}(\mathbf{u}(\tau),\tau) = [V'(\boldsymbol{\lambda},\tau)]\,\mathbf{u}^{\mathbf{T}}(\tau) + 1/2\mathbf{u}(\tau,\boldsymbol{\lambda})\,\mathbf{Hess}[\mathbf{V}(\boldsymbol{\lambda},\boldsymbol{\tau})]\,\mathbf{u}^{\mathbf{T}}(\tau)$.
Let $\mathbf{y}_{\mathbf{cr}}(t,\boldsymbol{\lambda},\mathbf{x})$ be solution of a system of equations:$\mathbf{y}_{\mathbf{cr}}(t,\boldsymbol{\lambda},\mathbf{x}_{\mathbf{cr}}) + \boldsymbol{\lambda} - \mathbf{x}_0 = 0$.We assume now that: for a fixed $t,\mathbf{x}_0$ and $\boldsymbol{\lambda}$ point $x_{\mathbf{cr}}(t,\boldsymbol{\lambda},\mathbf{x}_0)$ does not

focal point on the trajectory, given via solution of the linear boundary problem (3).Then for any Colombeau solution of the Schrödinger equation with initial data (2) the inequalities is satisfied: (4) $\liminf_{\hbar\to 0}\left|\langle i,t,\mathbf{x}_0;\hbar,\varepsilon\rangle-\lambda_i(t)\right|\leq$ $\left|\widehat{u}_i\left(t,\boldsymbol{\lambda}(t),\mathbf{x}_0\right)\right|$. We note that one recognized quantum trajectories $\mathbf{x}_q(t)=\left(x_1^q(t),...,x_n^q(t)\right)$ as limit $x_i^q(t)=\lim_{\hbar\to 0}\langle i,t,\mathbf{x}_0;\hbar,\varepsilon\rangle$ .By using inequalities (4) we obtain quantum trajectories as solution of a system of *master equations*: (5) $\widehat{u}_i\left(t,\boldsymbol{\lambda}(t),\mathbf{x}_0\right)=0, i=1,...,n$.Let us consider one dimensional quantum anharmonic oscillator with a cubic potential,supplemented by an additive sinusoidal driving i.e.$V(x,\tau)=m\omega^2x^2/2+ax^3-bx-\left[A\sin(\Omega\tau)\right]x$.In this case master equations reduces to a single equation (6) $d(\lambda)\left(\frac{\cos(\varpi t)}{\varpi}-\frac{1}{\varpi}\right)+$ $\frac{A\left[\varpi\sin(\Omega t)-\Omega\sin(\varpi t)\right]}{\varpi^2-\Omega^2}-\lambda m\varpi\cos(\varpi t)+m\varpi\cos(\varpi t)\,x_0=0, d(\lambda)=m\omega^2\lambda+$ $3a\lambda^2-b, \varpi(\lambda)=\sqrt{2\left(\omega^2/2+3a\lambda/m\right)}$, $\omega^2/2+3a\lambda/m\geq 0$ (see [2]).

References

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