Modeling the state transiton of the hippocampal local field potential between theta rhythm and large irregular amplitude activity by a bifurcation between limit cycle attractor and a chaotic dynamics

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Here we propose a possible mathematical structure of the state transition of the hippocampal local field potential (LFP) between theta rhythm and large irregular amplitude activity (LIA) in terms of nonlinear dynamics. The basic idea is that the alternation of the state between theta rhythm and LIA can be interpreted as a bifurcation of the attractor between limit cycle and chaos. Tsuda et al. reported that a network composed of simple class 1 model neurons connected with gap junctions show both asynchronous chaotic behavior and synchronous periodic behavior [1]. Here we model the network of hippocampal interneurons extending their model.
The network is composed of electrically coupled simple 2-demensionsl neurons with a natural resonant frequency in the theta range.

$$\frac{dx_i}{dt} = -y - \mu x_i^2 (x_i - \frac{3}{2}) + \mathrm{I} + \mathrm{g_{GJ}}(\mathrm{x_{i+1}} + \mathrm{x_{i-1}} - 2\mathrm{x_i})$$

$$\frac{dy_i}{dt} = -y - \mu x_i^2$$

We show that this model reproduces the characteristic appearance and disappearance of the spectral peak in the theta range. Because all the model neurons in the network have the same natural frequency in the theta range, the network dynamics naturally possesses a weekly unstable saddle periodic orbit corresponding to the all-synchronized state even with the parameter values of the chaotic regime. This intrinsic theta rhythm can be stabilized by an oscillatory ascending activity mimicking the medial septal projection and the synchronized state can be induced.

[1] I. Tsuda et al. J Integr Neurosci., 3:159-182. (2004).