DIMENSIONALITY REDUCTION OF DYNAMICAL SYSTEMS WITH PARAMETERS

John Brooke[1], Chris Welshman[2]

(1) School of Computer Science, University of Manchester, UK
(2) School of Mathematics, University of Manchester, UK

There is considerable interest in the idea that networks of very large numbers of neurons can exhibit dynamics that can be described by a manifold of very low dimension, and several of the talks at this meeting address this issue. We present a mathematical method that involves obtaining a low dimensional description of a system that is originally specified in high dimensional terms. Unlike classic methods of data reduction which consider data alone, we are attempting to reproduce the dynamics of the high dimensional system in a much lower dimensional ambient space so that it can be more easily studied. There are mathematical theorems (principally Whitney's embedding theorem) which guarantee that, if the high dimensional system produces dynamics which can be represented on a low dimensional manifold, the dynamics can be reproduced in a space of a dimension comparable to this manifold [1]. Models of dynamics often involve parameters representing physical processes, an example from neuroscience would be properties of the perceptual apparatus in considering networks that control vision and orientation. Thus we consider dynamical systems with parameters, where the parameters index a family of vector fields which produce a corresponding family of attractors. Existing methods of dimensionality reduction typically either do not consider the inclusion of parameters, or they deal with control inputs, where the focus is on preserving input-output behaviour, rather than geometric structures in the state space (e.g. of the neuronal networks). Although the attractors can change significantly with respect to the parameters, the underlying vector field is often smoothly dependent on the parameters. We can take advantage of this, to produce a low-dimensional family of vector fields that reproduce the corresponding attractors which are indexed by a parameterisation from the original parameter space. By focusing on retaining the relevant parts of the vector fields and allowing the attractors to emerge from their flows, bifurcations can even be reproduced. The approach consists of two steps: obtaining a dimension-reducing map that is suitable for the region of parameter space, and then using that map to find a parameter space map that describes a suitable family of vector fields.

[1] D. S. Broomhead, M. Kirby, Nonlinear Dynamics, **41**, 47-67 (2005).