MULTIFRACTAL ANALYSIS OF JULIA AND QUASI-JULIA SETS FROM NONLINEAR QUANTUM PROTOCOLS

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Fractal structures often emerge in the study of nonlinear dynamical systems, providing valuable insights into the complexity of underlying processes. In quantum information science, nonlinear quantum transformations play a significant role in quantum state distillation and quantum communication protocols. These transformations generally rely on post-selective measurements, which break the deterministic evolution of closed quantum systems, turning it probabilistic and nonlinear [1]. We study nonlinear protocols involving a single-qubit gate parametrized by a complex number p, CNOT gates, and single-qubit measurements on multiple identical input states [2, 3]. For pure quantum states, the protocol is described by complex rational maps on the Riemann sphere, generating Julia sets with rich fractal boundaries and self-similarity. At lower purities, it it gives rise to quasi-Julia sets, generalized fractals embedded in three-dimensional space. Through detailed numerical simulations, we characterize the resulting fractal structures by calculating their dimensions using box-counting and correlation dimension techniques. Our findings quantify how fractal dimensionality depends explicitly on the complex parameter p and the degree n of the rational map. We identify and explain a previously observed phase transition, governed by the purity of the initial quantum state. This transition marks a fundamental shift between non-fractal and fractal-like boundaries in the dynamical system. We further expand our analysis of the emerging fractal structures by investigating their multifractal characteristics.

[1] First reference citation here (to be filled with actual reference)
[2] Second reference citation here (to be filled with actual reference)
[3] Third reference citation here (to be filled with actual reference)

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