PHENOMENOLOGICAL RENORMALIZATION GROUP: A GEOMETRIC PERSPECTIVE

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The Critical Brain Hypothesis remains a significant challenge in both theoretical and experimental neuroscience. This hypothesis postulates that neuronal components operate as a collective system near a critical point. Such a premise has profound implications for our understanding of the brain and its functioning, with potential applications in medicine and technological advancements. To assess the validity of this hypothesis, various tools have been proposed to estimate, from experimental data, how close neural activity might be to a phase transition. Among these tools is the central technique of this study: the Phenomenological Renormalization Group (pRG). In 2019 [1], a group of researchers introduced a tool based on the Theoretical Renormalization Group to analyze the criticality hypothesis in time series of neuronal activity. Specifically, they applied this method to a population of more than 1000 neurons in the hippocampus of a mouse. The main idea behind this approach is to perform a coarse-graining of neuronal activity: at each step, the system is reduced to half of its neurons while attempting to retain invariant properties under this transformation, discarding those that might be redundant for information extraction. This technique opens the possibility of computationally and experimentally testing non-trivial scale invariance in statistical observables of interest.

On the other hand, in 2022, a study [2] put this procedure to the test. This analysis relies on empirical electrophysiological data, where the activity of thousands of individual neurons (specifically, the precise timing of their spikes) is simultaneously recorded at a high resolution of 200 Hz across multiple mouse brain regions. Using the pRG-based coarse-graining procedure across brain regions, this study produced results characteristic of systems near a phase transition, further intensifying the intrigue surrounding the Critical Brain Hypothesis.

In the present work, we aim to analyze the foundation of this Phenomenological Renormalization technique mainly from a theoretical and computational perspective. Our primary goal is to assess the reliability of this method in detecting criticality signals. Without intending to criticize previous studies, we aim to place this technique within a broader theoretical framework, enabling its generalization for further studies. This effort seeks to contribute to reducing biases that currently seem inherent to working with experimental data.

We have conducted preliminary tests of this technique using synthetic data generated by biologically motivated theoretical models, identifying limitations in its ability to reliably measure non-trivial scaling. In particular, we have observed challenges in distinguishing models that exhibit genuine phase transitions from those that merely display long but finite correlation times, comparable to the length of the time series. This motivates to extend the technique in order to analyze these biases. Thus, we have established a mathematical formalization of the coarse-graining process employed by the Phenomenological Renormalization Group, providing a rigorous geometric-framework for its application. Building on this formalization, we have extended the technique and demonstrated its connections with other innovative methodologies for dimensionality reduction and noise robustness, including Topological Data Analysis (PCA, UMAP, Persistent Homology, etc.). Finally, we have applied this approach to available experimental data on neuronal activity across different brain regions, obtaining results consistent with criticality signatures observed in previous studies.

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[1] Meshulam, Leenoy and Gauthier, Jeffrey L. and Brody, Carlos D. and Tank, David W. and Bialek, William, *Coarse Graining, Fixed Points, and Scaling in a Large Population of Neurons* (Phys. Rev. Lett. 2019).

[2] Guillermo B. Morales, Serena Di Santo, and Miguel A. Munoz, *Quasi-universal scaling in mouse-brain neuronal activity stems from edge-of-instability critical dynamics*, (Proc Natl Acad Sci U S A. 2022).