EFFECTIVE DIMENSIONAL REDUCTION OF MARKOVIAN SPREADING DYNAMICS ON COMPLEX NETWORKS

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The exact treatment of Markovian models on complex networks requires knowledge of probability distributions exponentially large in the number of nodes n. Mean-field approximations provide an effective reduction in complexity of the models, requiring only a number of phase space variables polynomial in system size. However, this comes at the cost of losing accuracy close to critical points in the systems dynamics and an inability to capture correlations in the system. In this talk, we introduce a tunable approximation scheme for Markovian spreading models on networks based on matrix product states (MPSs). By controlling the bond dimensions of the MPS, we can investigate the effective dimensionality needed to accurately represent the exact 2^n dimensional steady-state distribution. We introduce the entanglement entropy as a measure of the compressibility of the system and find that it peaks just after the phase transition on the disordered side, in line with the intuition that more complex states are at the "edge of chaos." The MPS provides a systematic way to tune the accuracy of the approximation by reducing the dimensionality of the system's state vector, leading to an improvement over second-order mean-field approximations for sufficiently large bond dimensions.

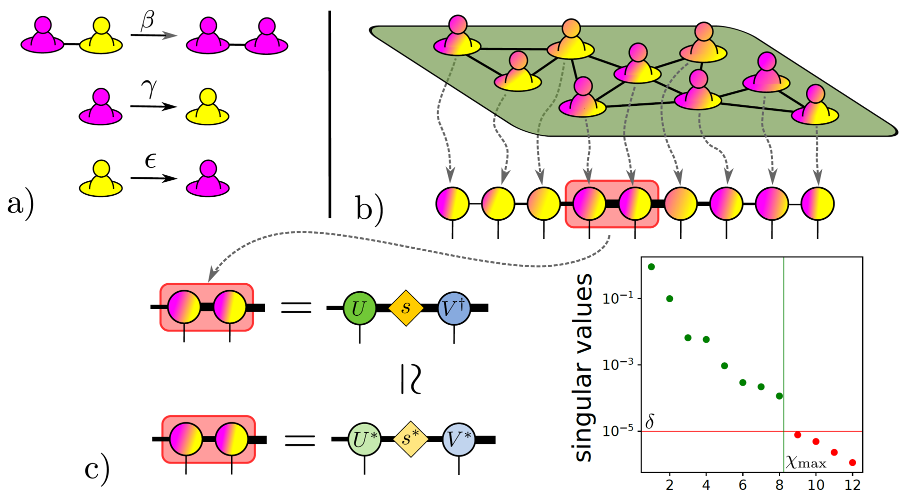

FIG. 1. Overview of the MPS approximation method. (a) The allowed transitions of the stochastic ϵ-SIS process: connected nodes can transmit an infection (purple) to their susceptible neighbors (yellow) with rate β, while they recover with rate γ. Susceptible nodes are spontaneously infected with rate ϵ. (b) The probability vector of a network state (top) is mapped to a matrix product state (MPS, bottom). The MPS consists of an array of tensors contracted over bond indices. The unconnected (physical) indices represent the two states the node may be in. (c) Each bond in the MPS is optimized by performing a singular value decomposition and then truncating the singular values below a threshold δ, or keeping a maximum of $\chi_{\max}$ singular values. The plot on the right shows the singular values of this particular bond, illustrating that we keep the largest singular values (in green) in the truncated diagonal matrix. In this way a compressed, lower-dimensional, approximation of the state vector can be obtained.

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