A NEW APPROXIMATE EASTIN-KNILL THEOREM

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Transversal encoded gatesets are highly desirable for fault tolerant quantum computing. However, a quantum error correcting code which exactly corrects for local erasure noise and supports a universal set of transversal gates is ruled out by the Eastin-Knill theorem. Consequently, every approach to building a useful quantum computer must, implicitly or explicitly, choose a strategy for circumventing this limitation. One such approach, is to relax the assumption of *exact* quantum error correction, by allowing for some (ideally) small error in the decoding process.

Here we provide a new approximate Eastin-Knill theorem for the single-shot regime when we allow for some probability of error in the decoding. In particular, we show that a quantum error correcting code can support a universal set of transversal gates and approximately correct for local erasure if and only if the conditional min-entropy of the Choi state of the encoding and noise channel is upper bounded by a simple function of the worst-case error probability. Our no-go theorem can be computed by solving a semidefinite program, and, in the spirit of the original Eastin-Knill theorem, is formulated in terms of a condition that is both necessary and sufficient, ensuring achievability whenever it is passed.

At it's heart, the Eastin-Knill theorem expresses an inherent tension between continuous symmetry principles and (exact) quantum error correction. To establish our no-go result, we leverage tools from asymmetry theory, a recent research line which aims to understand the constraints imposed by symmetries on general quantum dynamics beyond the traditional scope of Noether's theorem.

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