DIRAC-OPERATOR-DRIVEN CONSENSUS DYNAMICS: TOWARDS TOPOLOGICAL DISTRIBUTED OPTIMIZATION IN HIGHER-ORDER NETWORKS

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Distributed optimization is a key framework for enabling multiple agents to solve global problems using only local information and limited communication. It plays a central role in large-scale systems such as sensor networks, robotics, smart grids, and several systems where centralized solutions are often infeasible. Most existing algorithms operate over single-layer (pairwise) networks, which overlook the non-pairwise interactions common in real-world systems. In recent works, based on the diffusion dynamics driven by tensor Laplacians, we have extended distributed optimization to multiplex and multilayer networks. Following these ideas, in this talk, based on the topological Dirac equation, we introduce a novel approach to distributed optimization on higher-order networks by leveraging Dirac operators — algebraic constructs rooted in discrete exterior calculus that unify node-based and edge-based dynamics under a common formalism. Building on recent advances in topological signal processing and higher-order network theory, we propose a class of Dirac-operator-driven consensus dynamics capable of operating over generalized complexes. This formulation enables agents not only to reach agreement over values associated with nodes, but also to coordinate over interactions involving edges, triangles, and higher-dimensional simplices. Our framework opens the door to topology-aware distributed algorithms that are sensitive to the underlying multi-scale structure of complex systems.

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