# EQUATIONS OF MOTION FOR ENGINES WITH THERMAL FEEDBACK

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Mathematical treatments of active systems are usually based on anti-damping, nonconservative positional forces, time delays, Maxwell demons, or other descriptions whose physical basis is often obscure. Seeking a more realistic description, we work out Newtonian equations of motion for the piston of a $\beta$-type Stirling heat engine. The piston is treated as a macroscopic mechanical degree of freedom with inertia, acted upon by a force that depends on the thermodynamic state of a working fluid. This force can do net positive work over a cycle because of a thermal feedback: the position of the piston controls the coupling of the working substance to an external thermal disequilibrium, here represented as two baths at different temperatures. This feedback replaces the linear anti-damping of the van der Pol or Stuart–Landau models, facilitating a more physical understanding of how non-linearities stabilize (against thermal noise) the frequency of the system running near its limit cycle. We argue that such a treatment may clarify the thermodynamic cost of biochemical clocks and other active cycles.

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